Girls Get
Curves

## Proof that the sum of every triangle's angles equals 180°!

Remember on p. 104 how we talked about there being 180° in the three angles of any triangle? Get this – now we can prove it! Here's the theorem:

If the angles of a triangle are added together, the sum will equal 180°.

Seems strange at first, like, how on Earth could we prove something like that?? Just think, in a few minutes you'll be like, "Oh yeah, no sweat." Aren't you impressed with your future self? I certainly am!

To prove that the three angles of all triangles add up to 180°, we're going to start with any 'ol triangle $\triangle ABC$, and mark its angles with ♥, ☺, and ★. Our goal will be to prove that ♥ + ☺ + ★ = 180°. If we do that, we're golden.
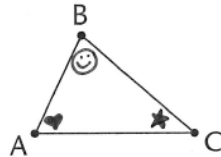
Take a look at the drawing below. On top of this regular 'ol triangle, we'll extend all three of its sides to lines, $\overleftrightarrow{AC}$ along the bottom, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ (in dotted lines), and draw a line parallel to $\overleftrightarrow{AC}$, going through the point B.[1] Don't worry about why we're doing this yet; just see if you can follow along… it'll be awesome.
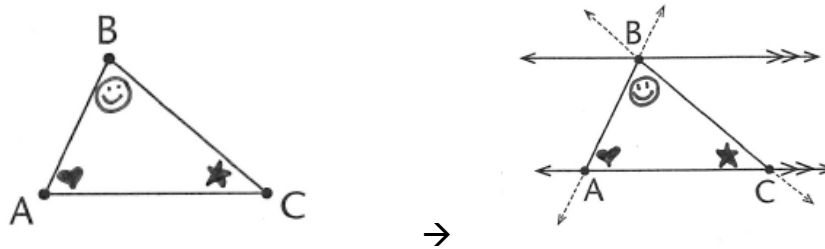

All we've done is extend lines from segments that were already drawn, and then we created a whole new line on top, one that is parallel to $\overleftrightarrow{AC}$. Notice that we've drawn in a total of 4 lines on top of our original triangle drawing. And two of those lines are actually transversals (the dotted ones), crossing two parallel lines (the solid ones). Take a moment to look at that middle drawing and make sure you're with me so far. Got it? Great!

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[1] There something called the "Parallel Line Postulate" that says such a line must be *unique*, so we can rest assured there wasn't some "better" line we could have drawn. Nothing to worry about, but see p.389 in *Girls Get Curves* if you want to read about it.

Okay, so what can we do now? Let's start filling in what we know, based on what we've learned with transversals (Remember escalators at the mall from chapters 13 & 14?). For a moment, let's pretend nothing else exists except the transversal (escalator) $\overleftrightarrow{AB}$ and the two parallel lines.

Because of Alternate Interior Angles being congruent, (look for the "Z"!), we can fill in the ♥ for the angle to the left of B, can't we? We know that no matter what the measure of ♥ is on the bottom left of the triangle, that alternate interior angle above must also have the same measure!
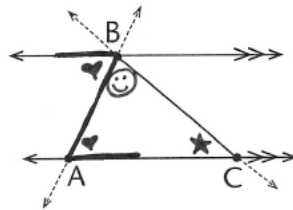

And we can do the same thing with the other transversal: Pretending nothing exists except the parallel lines and the transversal $\overleftrightarrow{BC}$, we know that the angle to the right of B must also be ★, the same as the bottom right angle of our triangle.
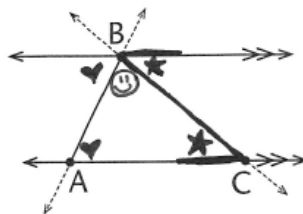
Look at all the stuff we've filled in! Nice. Now let's step back for a moment and remember that a straight angle measures 180°, so it makes sense that if we divide a straight angle into smaller angles, the smaller angles must add up to 180°. (See the diagram below, which is just the top part of the diagram above.)
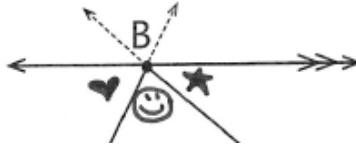
Notice it must be true that: ♥ + ☺ + ★ = 180°. And since those three angles are congruent to the three angles in our triangle, we've proven that the degrees in our triangle add up to 180, too! ♥ + ☺ + ★ = 180°.
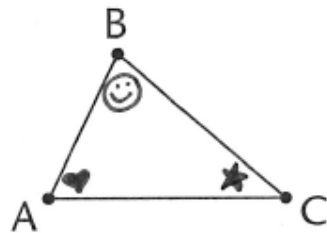
♥ + ☺ + ★ = 180°

How about that!

Notice that since there was nothing "special" about our triangle – in other words, we started with a totally generic triangle, and we didn't need anything other than the characteristics of a generic triangle in order to do this proof. That means we've proven it for *all* triangles.

Again, don't worry if you wouldn't have known how to create this proof on your own! But it's a great idea for you to try it on your own, now that you've seen how it works. I like this proof because it feels like we've tricked the diagram somehow - like we've cornered it into telling us what we want to know.

Imagine if the triangle were playing hard to get. Can't you just see the scene?

Us: "Your angles add up to 180°, don't they?"

Triangle: "I'll never tell!"

Us: (innocently) "Well…you wouldn't mind if we just draw a little line through point B, parallel to the opposite side, would you? And extend your sides into lines?"

Triangle: "Sure, knock yourself out."

(We draw these in.)

Us: "Hm, it appears that we have some Alternate Interior Angles now - which means we have some congruent angles we can fill in. You certainly wouldn't mind if we do that, would you?"

Triangle: "Uh… I guess not."

(We fill in the ♥ and ★.)

Us: "Isn't it true that 180 degrees make up a straight line?"

Triangle: "Well, yes, obviously."

Us: "Then I guess you can't argue that ♥ + ☺ + ★ = 180°."

Triangle: "Well, no, but what does that have to do with my angles? Oh, wait…"

And then we win, don't we?

Seems kind of cool and sneaky to me, anyway. And this is really what we're doing every time we prove something - we're making it impossible for the other side to win! That's why our logic has to be airtight…

Notice that the key in doing the above proof was to draw stuff in! We extended lines and created a set of parallel lines in order to create the escalator-at-the-mall drawing, and then we were able to apply what we know about transversals. This stuff is sinking in, I promise!