The Commutative Property for Transformations

...and some serious phone-flipping action

(As promised on p. 117 of Girls Get Curves)

Remember the commutative property? ¹

It says things like $a + b$ is the same as $b + a$. The order doesn’t matter; we end up with the same result (answer) either way. This might seem obvious for something like addition, but what about for transformations?

Something I’ve always found interesting about compositions of transformations is that sometimes the order matters, and sometimes it doesn’t. So if we rotate a wheel (say, clockwise) $20^\circ$, and then rotate it another $35^\circ$, it’s the same as if we first rotated it $35^\circ$ and then $20^\circ$ – if we’re always rotating clockwise, then either way, our wheel will be rotated $55^\circ$ clockwise from where we started. No biggie. Or if we glide an object 20 ft in a particular direction and then another 35 ft, that’s the same as if we did 35 ft and then 20 ft. This is all pretty obvious, you say, because $20 + 35 = 35 + 20$, right?

¹ Check out chapter 2 in Kiss My Math for more on commutative properties and my “cute poem” challenge.
But compositions of transformations aren’t necessarily commutative… Here’s an example: Take a phone, or a book, and hold it up in front of you.

First flip it towards yourself over an imaginary horizontal line, and then rotate it clockwise 90°. You’re now looking at the back of it, right? Memorize something you see on the right side (below, it’s a heart).

First flipped over horizontal line, then rotated clockwise 90°

Now, hold it in front of you again, in the same starting position as before. But this time, first rotate it clockwise 90°. You’re still looking at the front, right? Now flip it towards you over that imaginary horizontal line. Lo and behold, that thing you noticed on the right is now on the left!

First rotated clockwise 90°, then flipped over horizontal line
Pretty wild, huh? And just like that, we can say that the composition of transformations isn’t necessarily commutative. Just kind of interesting, that’s all!