

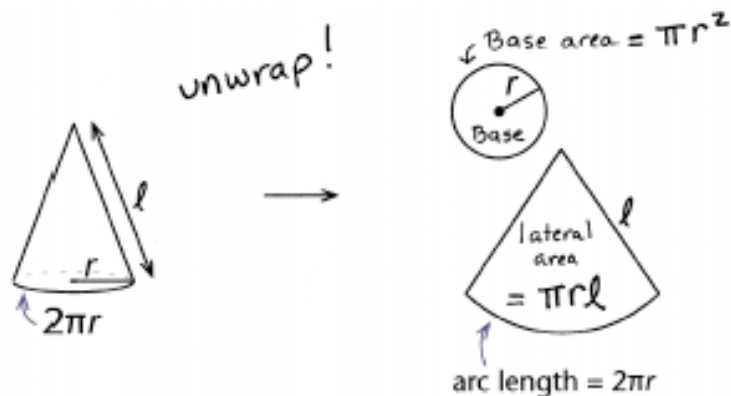
Girls Get Curves

Surface Area of a Cone Formula:

Where Does the “Fan” Part Come From?

(As promised on p. 370 in *Girls Get Curves*)

On p. 370 of *Girls Get Curves*, we saw that the formula for the surface area of a cone is just the area of the base (which is a circle: πr^2) plus the area of the “fan” shape, which turns out to be πrl , where l is the slant height of the original cone.



$$\begin{aligned} \text{Total S.A. of a Cone} &= \text{area of Base} + \text{area of fan} \\ &= \pi r^2 + \pi rl \end{aligned}$$

We also talked about how since π is spelled *pi*, the “fan” part of this cone formula sort of spells “pirl”, which makes me think of a fan made of pearls.

That’s a nice, easy way to remember the formula, but where does this “fan” part of the formula come from, anyway? Let’s find out!

Notice that the fan is actually a sector (slice) of a big circle¹, with radius l . Let’s imagine this big circle that this slice is a part of. Since its radius is l , that means its total circumference would be $2\pi l$, right? And its total area would be πl^2 . Not so bad, right?

Next, we need to figure out what *fraction* of the entire circle this slice is. Since the slice’s arc length is $2\pi r$ (look at the diagram to remember why), and the total big circle’s arc length (in other words, its circumference) is $2\pi l$, then here’s the ratio of our slice to

the big circle, expressed as a fraction: $\frac{\text{slice}}{\text{total circle}} = \frac{2\pi r}{2\pi l} = \frac{r}{l}$.

Now that we have this ratio, we can take the total area of the circle, multiply it by this fraction, and get the area of just the slice! (In other words, the area of our fan!)

$$\text{Area of slice} = \frac{r}{l} (\text{area of circle})$$

And now we can fill in the values we know:

$$\rightarrow \text{Area of slice} = \frac{r}{l} \cdot \pi l^2$$

$$\rightarrow \text{Area of slice} = \frac{\pi r l^2}{l} \quad (\text{now we cancel an “}l\text{” from top and bottom})$$

$$\rightarrow \text{Area of slice} = \pi r l$$

¹ Okay, technically we can’t just “notice” this – but we’ll take it for granted for this proof. In higher math, you’d be expected to prove that part, too. Basically, because of the symmetry of a right circular cone, the unwrapped “fan” part is forced into having a uniform curvature... which, along with its equal sides (both are l), this forces it into being the sector of the circle with radius l .

Ta-da!

Yep, since the “slice” is our fan, we’ve just proven that the “pearl fan” part of a cone’s surface area formula is $\pi r l$. We already knew the area formula for the base of a cone with radius r ; it’s just πr^2 . So now we see where the entire formula comes from. Nice!



$$\begin{aligned} \text{Total S.A. of a Cone} &= \text{area of Base} + \text{area of fan} \\ &= \pi r^2 + \pi r l \end{aligned}$$