Girls Get Curves

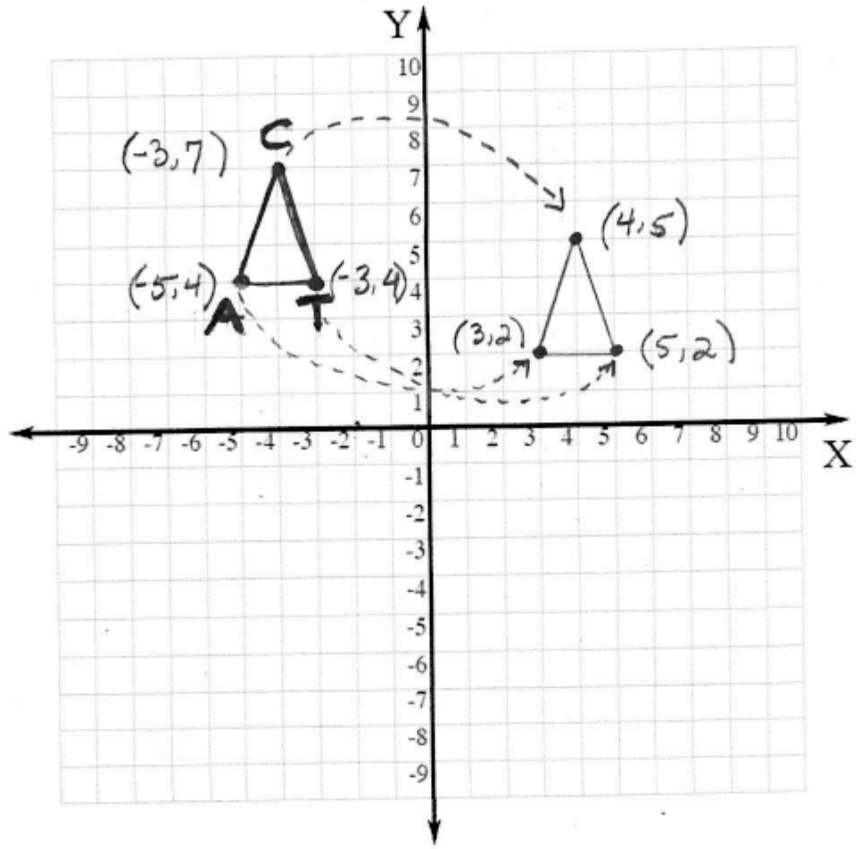
Coordinate Geometry for Transformations – Free Worksheet!

(As promised on p. 117 of *Girls Get Curves*)

Remember the good 'ol coordinate plane? Plotting points and lines, and all that good stuff? I know how dearly you've missed it. Well, it's easy to do precise **transformations** when we do them on the coordinate plane, because we know exactly where everyone is, and where they're going!

Let's say we wanted to slide the below triangle $\triangle CAT$ to the right 8 units and down 2 units. Then every point of the shape would need to make that move,[1] right? If the three vertex points of $\triangle CAT$ are $C(-3, 7)$, $A(-5, 4)$ and $T(-3, 4)$, we just add 8 to each x-value – that'll move the points to the right 8 spots on the grid – and subtract 2 from each y-value – that'll move 'em all down 2 spots on the grid.

[1] All we have to do is move all *vertex* points of our shapes, and then connect the lines.



Ta-da! Successfully translated. In other words, we've taken each vertex (x, y), and changed it like this: (x + 8, y – 2). Make sense? When we do the translation correctly, the shape should be congruent to the original shape – it'll just be in a different spot!

There are lots of ways to do congruence transformations on the coordinate plane, but there are a few particularly popular ones, and it's nice to be familiar with them. Here they are:

SHORTCUT ALERT

Use these shortcuts to make transformations on the grid a snap!

Translation: Each point moves a units in the x- direction and b units in the y- direction: $(x, y) \rightarrow \mathbf{(x + a, y + b)}$[2]

Reflection across the x-axis: Each x-value stays the same, and each y-value becomes the opposite of what it was: $(x, y) \rightarrow \mathbf{(x, -y)}$

Reflection across the y-axis: Each y-value stays the same, and each x-value becomes the opposite of what it was: $(x, y) \rightarrow \mathbf{(-x, y)}$

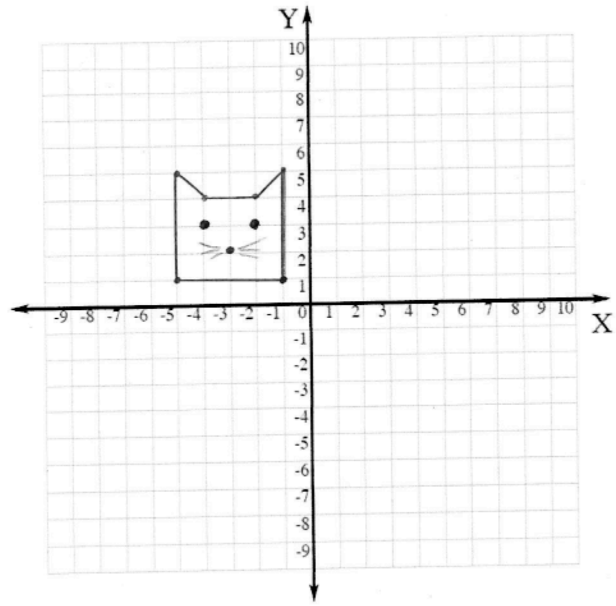
Reflection across the line $y = x$: The x- and y-values switch spots! $(x, y) \rightarrow \mathbf{(y, x)}$

Rotation about the origin: Each x- and y-value becomes the opposite of what it was: $(x, y) \rightarrow \mathbf{(-x, -y)}$

This stuff makes more sense when see it in action…

[2] Remember, *a* and *b* could be negative, and "adding a negative" is the same as subtraction. To review this really important concept, check out chapter 1 in *Hot X: Algebra Exposed*, especially pp. 6-8.

Siamese Triplets?!

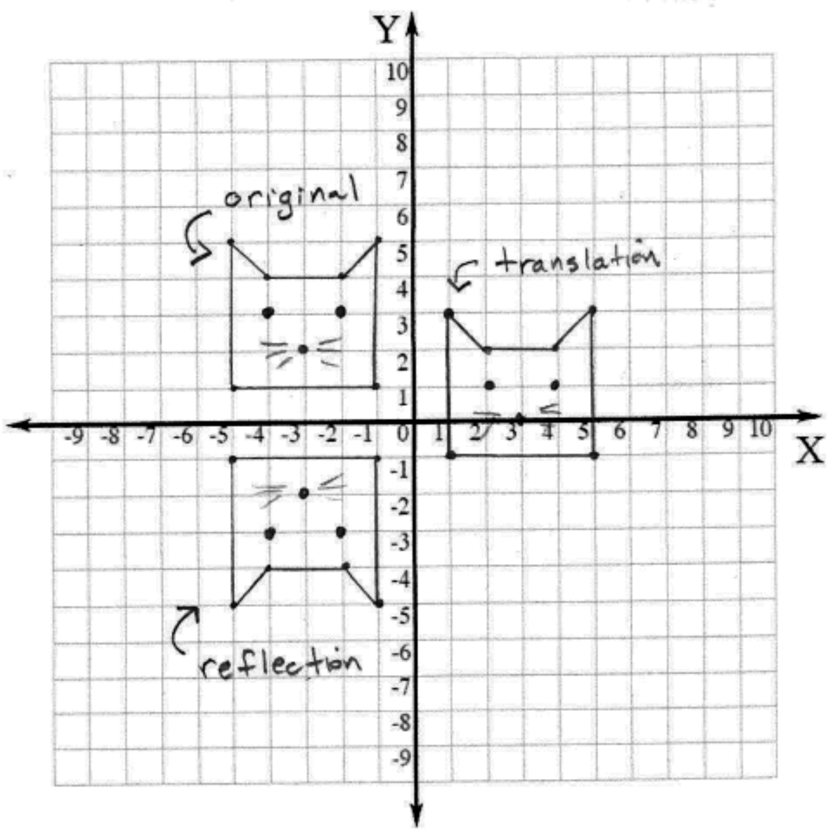


Let's try some of this coordinate transformation stuff for ourselves. This cat face (not counting the eyes and nose) "vertex" points are (–5, 1), (–5, 5), (–4, 4), (–2, 4), (–1, 5), (–1, 1). We'll do two different transformations on this shape, resulting in three identical (congruent) kitties. We'll do a reflection (flip) and a translation (glide)![3]

<u>Reflection across *x*-axis</u>: $(x, y) \rightarrow (x, -y)$	**<u>Translation</u> down 2 units, and to the right 6 units: $(x, y) \rightarrow (x + 6, y - 2)$**
The *x*-values all stay the same; the *y*-values (the heights) all become the negative of what they were:	Every point gets moved down and to the right:
Old cat's pts…………New cat's pts	Old cat's pts……………New cat's pts
(–5, 1) → **(–5, –1)**	(–5, 1) → (–5 + **6**, 1 – **2**) = **(1, –1)**
(–5, 5) → **(–5, –5)**	(–5, 5) → (–5 + **6**, 5 – **2**) = **(1, 3)**
(–4, 4) → **(–4, –4)**	(–4, 4) → (–4 + **6**, 4 – **2**) = **(2, 2)**
(–2, 4) → **(–2, –4)**	(–2, 4) → (–2 + **6**, 4 – **2**) = **(4, 2)**
(–1, 5) → **(–1, –5)**	(–1, 5) → (–1 + **6**, 5 – **2**) = **(5, 3)**
(–1, 1) → **(–1, –1)**	(–1, 1) → (–1 + **6**, 1 – **2**) = **(5, –1)**

[3] *Note: this is not a composition of transformations; we're just doing two different transformations that happen to be on the same graph.*

The columns above might remind you of something: functions. Remember our good 'ol sausage factories[4]? We take inputs, put 'em in the sausage factory (function), and we get ouputs. Here, instead of single numbers, the inputs are the points (both the *x*- and the *y*- values) from the original shape, and the outputs are the points (both the *x*- and the *y*- values) of the image. And instead of an equation for a line, the "function" is the *transformation itself*. Here are the new cat faces (with our new coordinate points), graphed below:



If the *x*-axis is the surface of a pond or a lake, then it looks to me like the kitty first saw herself in the water's reflection, and then decided to have a little sip!

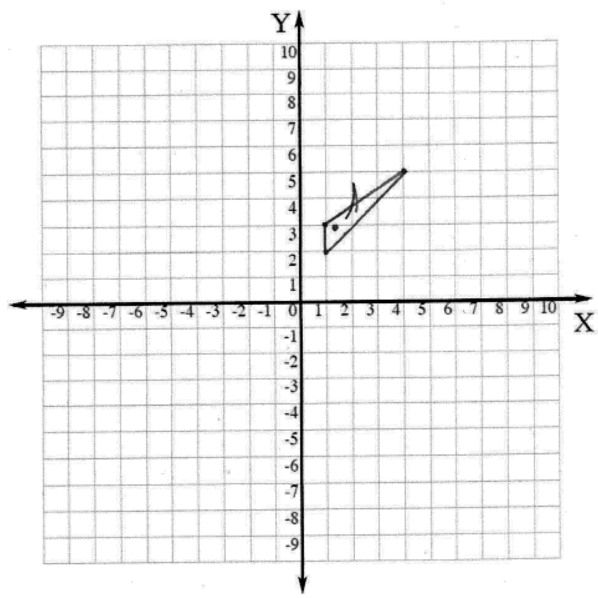
---

[4] To review functions (and sausage factories) check out chapter 17 in *Kiss My Math*.

Oh hey, speaking of ponds and lakes, let's practice this stuff by moving a scalene triangle fish[5] around on the grid, shall we?

DOING THE MATH

The scalene triangle fish to the right is defined by the three points (1, 2), (1, 3), and (4, 5). Perform transformations on this shape based on the rules given. I'll do the first one for you.



Part a: List the new coordinates.

Part b: Draw the new graph (eyes and fins optional!), along with the original fish.

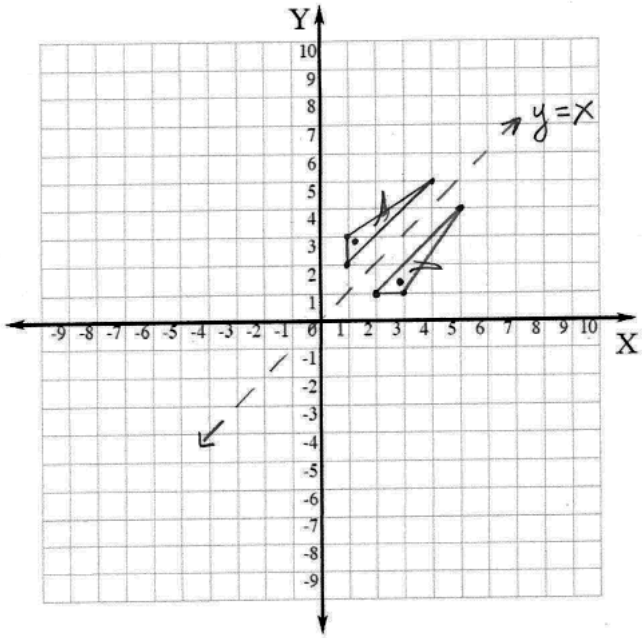
Part c. Describe the transformation.

1. Transform: $(x, y) \rightarrow (y, x)$

<u>Working out the solution</u>: We don't have to remember from the shortcuts which transformation this is; we can just apply this rule and see what the new coordinates will be – and discover it! Looks like we're just supposed to the spots of *x* & *y*, and we get: (1, 2) → **(2, 1)**; (1, 3) → **(3, 1)**; (4, 5) → **(5, 4)**. Done with part a! Next, let's plot these points (2, 1), (1, 3), and (5, 4) and see what we get.

---

[5] Get it? *Scale*ne, like scales on a fish? I know, I'm sooo clever, right?



Hm, it doesn't look like a translation (glide), and we couldn't have gotten to the new fish's spot by putting our fish on a bicycle wheel, so it's not a rotation. The fish seems to be looking in the mirror! So it's a reflection. But which mirror? The mirror created by the line $y = x$!

Answer:

a. (2, 1), (1, 3), and (5, 4)

b. (see graph above)

c. It's a reflection across the line $y = x$.

2. Transform the fish: $(x, y) \rightarrow (x + 3, y - 6)$

3. Transform the fish: $(x, y) \rightarrow (-x, y)$

4. Transform the fish: $(x, y) \rightarrow (x, -y)$

5. Transform the fish: $(x, y) \rightarrow (-x, -y)$

6. Transform the fish: $(x, y) \rightarrow (y, -x)$

7. Transform the fish: $(x, y) \rightarrow (-x, y + 2)$

In the above exercises, #6 and #7 are examples of how we can put *two transformations together* to make a whole new transformation – a **composition of transformations** – just like we saw on p. 117 of *Girls Get Curves*.

Answer Key and Solution Guide:

2. We start out with a fish whose coordinates are (1, 2), (1, 3), and (4, 5), right? And now we're supposed to find the new coordinates using this rule: $(x, y) \rightarrow (x + 3, y - 6)$, so we'll get:

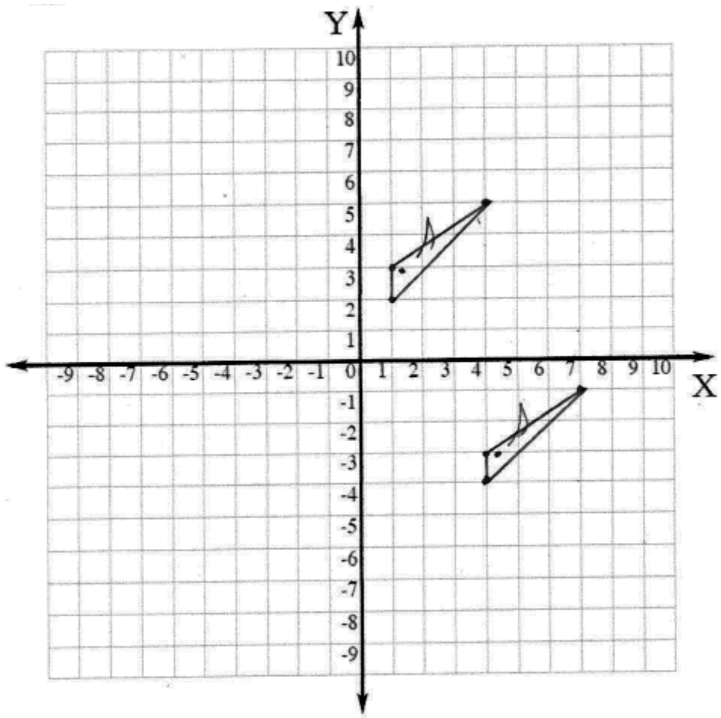
$(x, y) \rightarrow (x + 3, y - 6)$:

$(1, 2) \rightarrow (1 + 3, 2 - 6) =$ **(4, –4)**

$(1, 3) \rightarrow (1 + 3, 3 - 6) =$ **(4, –3)**

$(4, 5) \rightarrow (4 + 3, 5 - 6) =$ **(7, –1)**

Now let's plot the points and draw our new fish!



Hm, how do we describe this transformation? Well, it's still facing the same way as before – it just glided over and down. Yep – it's **a translation**! (We also could have noticed this from the Shortcut Alert on p. 2 of this worksheet.)

Answer:

Part a. (4, –4), (4, –3) and (7, –1)

Part b. See above graph

Part c. It's a translation

3. Okay, so we'll find the new coordinates using this rule: $(x, y) \rightarrow (-x, y)$, so we get:

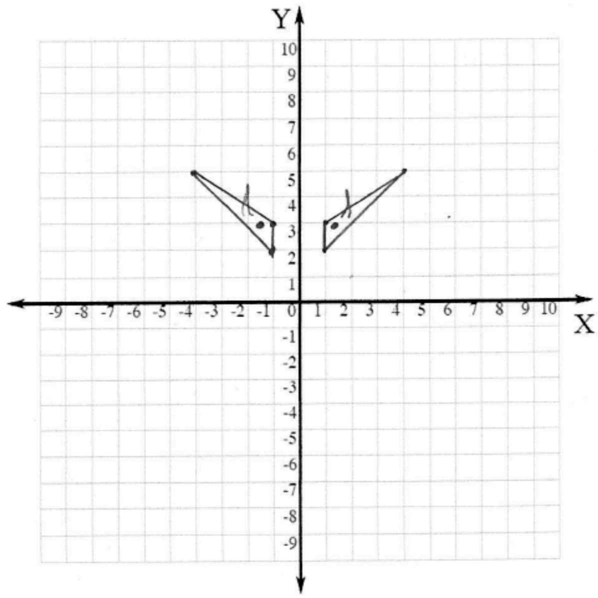
$(x, y) \rightarrow (-x, y)$:

(1, 2) → **(–1, 2)**

(1, 3) → **(–1, 3)**

(4, 5) → **(–4, 5)**

…and we plot three new points to find out where our fish has gone now…



How do we describe this transformation? Well, notice that the *y*-axis is like a mirror, and the fish is being reflected in it. Yep! It's **a reflection across the *y*-axis**. (We also could have noticed this from the Shortcut Alert on p. 2 of this worksheet.)

Part a. (–1, 2), (–1, 3), (–4, 5)

Part b. See above graph

Part c. It's a reflection across the *y*-axis

4. We'll find this fish's new coordinates using this rule: $(x, y) \rightarrow (x, -y)$ and get:

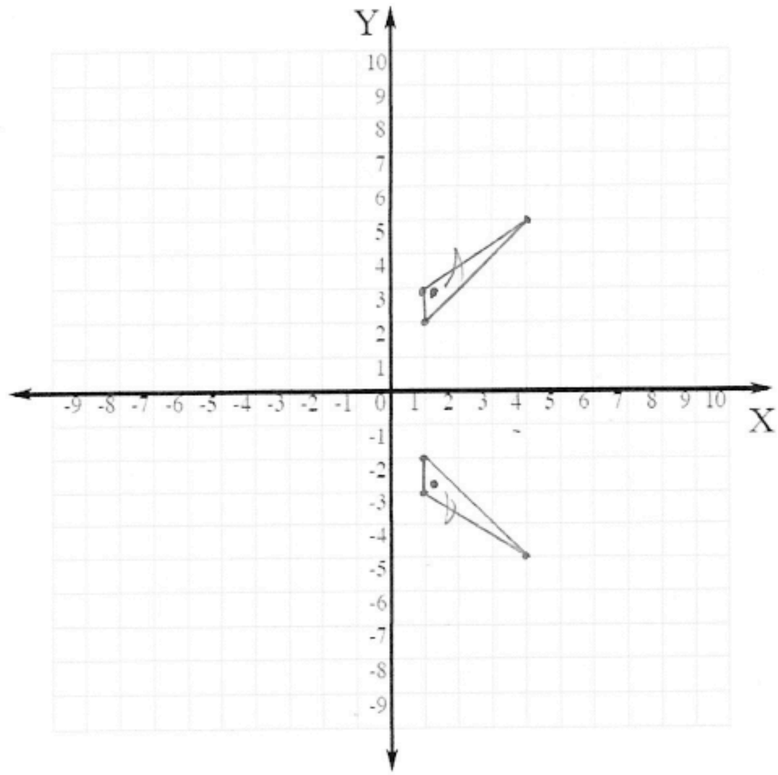
$(x, y) \rightarrow (x, -y)$

(1, 2) → **(1, –2)**

(1, 3) → **(1, –3)**

(4, 5) → **(4, –5)**

…and now we'll plot three new points to find our new fish!



And how do we describe this transformation? This time, the *x*-axis is like the mirror that the fish is being reflected in, so it's **a reflection across the *x*-axis**. (We also could have noticed this from the Shortcut Alert on p. 2 of this worksheet.)

Part a. (1, –2), (1, –3), (4,– 5)

Part b. See above graph

Part c. it's a reflection across the *x*-axis

5. Let's find our fish's new coordinates using this rule: $(x, y) \rightarrow (-x, -y)$ and we get:

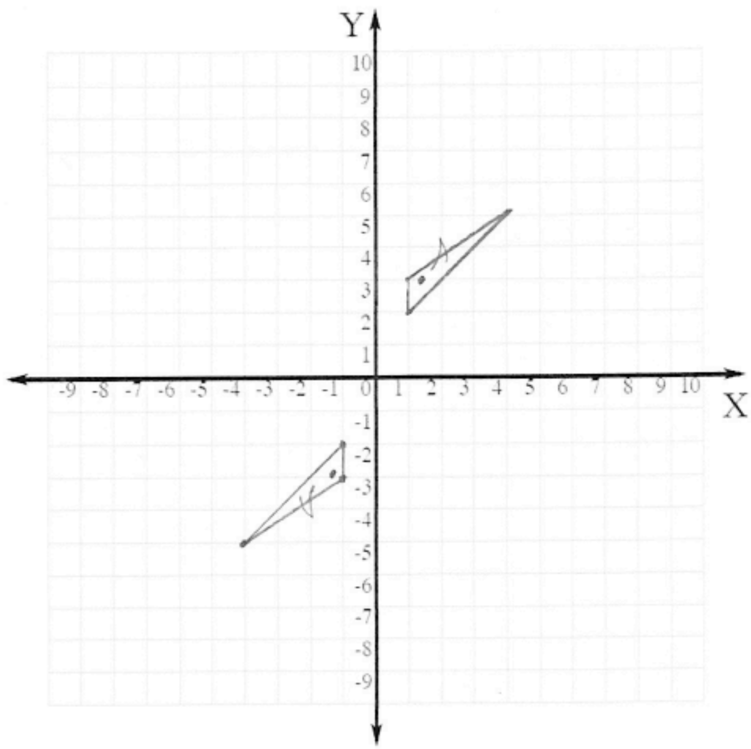
$(x, y) \rightarrow (-x, -y)$

$(1, 2) \rightarrow$ **$(-1, -2)$**

$(1, 3) \rightarrow$ **$(-1, -3)$**

$(4, 5) \rightarrow$ **$(-4, -5)$**

…and now we'll plot three new points to find our new fish!



How do we describe this transformation? Imagine that the origin (0, 0) is the center of a bicycle wheel that we rotate 180°. Then you can see that our new fish is just the old fish, rotated about the origin – so this transformation is a **rotation about the origin**. (We also could have noticed this from the Shortcut Alert on p. 2 of this worksheet.)

Part a. (–1, –2), (–1, –3), (–4, –5)

Part b. See above graph

Part c. it's a rotation about the origin

6. Let's find our fish's new coordinates using this rule: (x, y) → (y, –x).

So first we switch the x and y spots, and then we make the second coordinate negative, so we get:

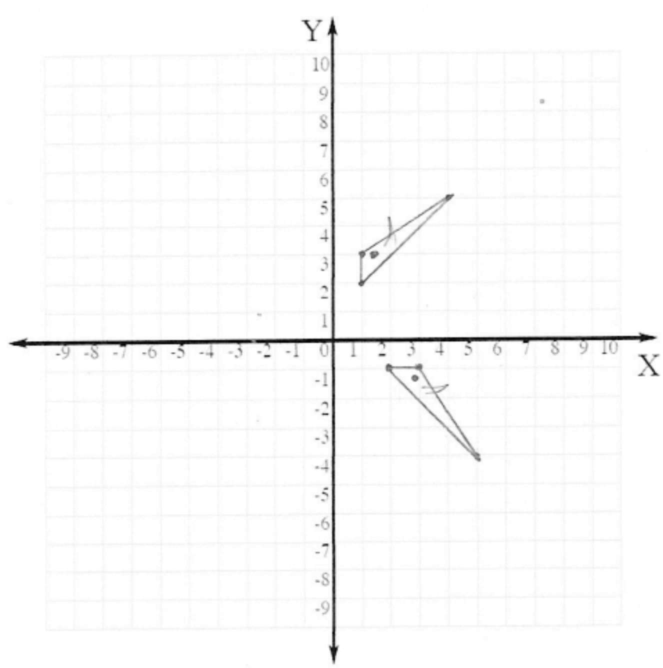
(x, y) → (y, –x)

(1, 2) → **(2, –1)**

(1, 3) → **(3, –1)**

(4, 5) → **(5, –4)**

…and now we'll plot three new points to find our new fish!



How do we describe this transformation? Well there are a few ways – this type of transformation isn't listed in the Shortcut Alert. One way to get from the original to the new fish, we could rotate it clockwise approx. 90°, and then glide it down and to the right. Or we could do the gliding down/right first, and then rotate it clockwise.

Part a. **(2, –1), (3, –1), (5, –4)**

Part b. See above graph

Part c. glide then rotate, or rotate then glide

7. Let's find our fish's new coordinates using this rule: $(x, y) \rightarrow (-x, y + 2)$

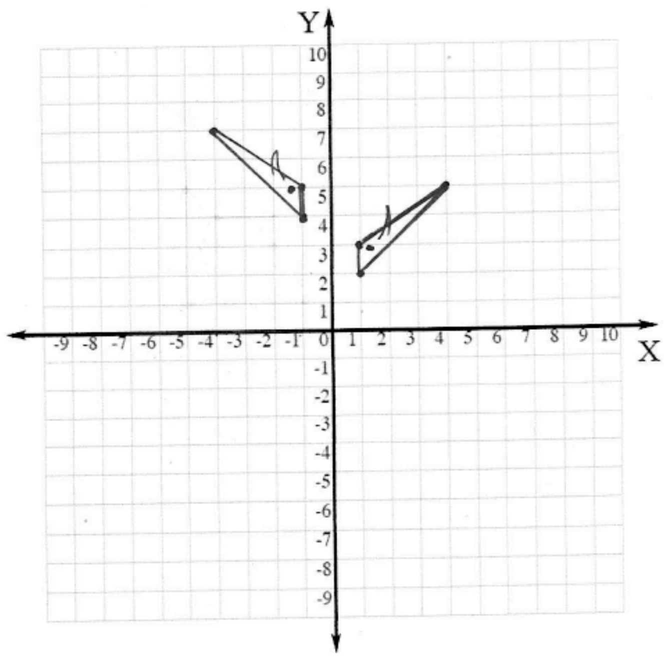
$(x, y) \rightarrow (-x, y + 2)$

$(1, 2) \rightarrow (-1, 2 + 2) \rightarrow$ **(–1, 4)**

$(1, 3) \rightarrow (-1, 3 + 2) \rightarrow$ **(–1, 5)**

$(4, 5) \rightarrow (-4, 5 + 2) \rightarrow$ **(–4, 7)**

And now we'll plot these points and see where the fish ends up this time…



How can we describe this transformation? We could first slide the original fish up two units and then reflect it over the *y*-axis, and we'd end up with the new fish, right? This is actually called **a glide reflection**, just like we saw on p. 117 of *Girls Get Curves* with our footprints on the beach!

Part a. (–1, 4), (–1, 5), (–4, 7)

Part b. See above graph

Part c. It's a glide reflection