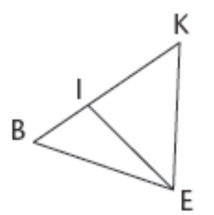
Girls Get Curves

Solution Guide for Chapter 10

Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

DTM from p. 169-170

2. Refer to BIKE. Given: $\overline{BE} \cong \overline{KE}$, I is not the midpoint of $\overline{BK}$. Prove: $\overrightarrow{EI}$ does not bisect $\angle BEK$.



In a proof by contradiction, we always start by assuming the *opposite* of what we're supposed to prove, right? A ray either bisects an angle or it doesn't – it's gotta be one or the other. So we'll assume that $\overrightarrow{EI}$ *does* bisect $\angle BEK$. And now let's see where that assumption leads us! So if $\overrightarrow{EI}$ bisects $\angle BEK$, then by the definition of "bisect," $\overrightarrow{EI}$ would split $\angle BEK$ into two congruent angles: $\angle BEI \cong \angle KEI$. (Gimmie an "A!") Putting that together with a given ($\overline{BE} \cong \overline{KE}$ - Gimmie an "S"!), and the third side that we get from the Reflexive property ($\overline{EI} \cong \overline{EI}$ - Gimmie an "S"!), we would have SAS congruency which would prove that $\triangle BEI \cong \triangle KEI$. And then by CPCTC, it would *have* to be true that $\overline{BI} \cong \overline{IK}$. But if *that's* true, then by definition, *I* would *have* to be the midpoint of $\overline{BK}$. And this is a direct contradiction to one of the Givens (*I* is not the midpoint of $\overline{BK}$)! And that means our assumption must have been wrong in the first place. We've now proven that the ray $\overrightarrow{EI}$ cannot bisect $\angle BEK$. Ta-da!

$\therefore \overrightarrow{EI}$ **does not bisect** $\angle BEK$

And here it is, in two-column form:

♥Proof♥

Statements	**Reasons**
1. $\overline{BE} \cong \overline{KE}$	Given (Gimmie an "S"!)
2. *I* is not the midpoint of $\overline{BK}$	Given
3. Assume $\overrightarrow{EI}$ bisects $\angle BEK$	Assumption leading to a possible contradiction
4. $\angle BEI \cong \angle KEI$	Definition of bisect (Gimmie an "A"!)
5. $\overline{EI} \cong \overline{EI}$	Reflexive property (Gimmie an "S"!)
6. $\triangle BEI \cong \triangle KEI$	SAS (1, 4, 5)
7. $\overline{BI} \cong \overline{IK}$	CPCTC
8. *I* is the midpoint of $\overline{BK}$	Definition of midpoint. Contradicts #2.
9. $\therefore \overrightarrow{EI}$ does not bisect $\angle BEK$	Our assumption ($\overrightarrow{EI}$ bisects $\angle BEK$) must have been false, because Statements #2 & #8 contradicted each other.

3. *Prove that there is an infinite number of negative numbers.*

Well, there's either a *finite* number or an *infinite* number of negative numbers, right? It's gotta be one or the other. So let's assume there is a finite number of negative numbers. If there's a *finite* number of negative numbers, then there would have to be a *lowest* negative number, right? Let's call that lowest negative number "*L*."

But then $L - 1$ would also be negative, wouldn't it? (Subtracting 1 always means we move left one place on the number line.) And $\boldsymbol{L - 1 < L}$, for all *L*. (Think about that for a moment – stick ANY real number in for *L* and you'll get a true statement!) So we've just found a negative number *lower* than *L*, haven't we? And that, missy, contradicts the assumption that it was the lowest one! That means our assumption must have been incorrect – *there can be no lowest negative number*, because we can always find one lower. And since the only option to our (now proven false) assumption is that there is indeed an *infinite* number of negative numbers, we've finished our proof!

$\therefore$ **There is an infinite number of negative numbers.**

4. Refer to SWIM. Given: $\overrightarrow{SI}$ bisects $\angle WSM$, $\angle SIW$ is obtuse.

Prove: $\overline{SW} \ncong \overline{SM}$. (Hint: First "prove" two triangles are congruent, and then figure out how to contradict "$\angle SIW$ is obtuse" by proving something about $\angle SIM$ & $\angle SIW$. You can do this!) NOTE: You can assume that $\angle WIM$ is a straight angle.

Either $\overline{SW} \cong \overline{SM}$ or $\overline{SW} \ncong \overline{SM}$; it's gotta be one or the other, right? Since we're supposed to prove $\overline{SW} \ncong \overline{SM}$, we'll start by assuming the opposite, $\overline{SW} \cong \overline{SM}$, and see if we can get a contradiction! The hint says to start by "proving" two triangle are congruent, so we'll keep that in mind…

So, if $\overline{SW} \cong \overline{SM}$, then two congruent segments means "Gimmie an "S"!" Also, since $\overrightarrow{SI}$ bisects $\angle WSM$, we also know that $\angle WSI \cong \angle MSI$ ("Gimmie an "A"!) Those two items, along with the good ol' reflexive $\overline{SI} \cong \overline{SI}$ (Gimmie an "S"!) would tell us that, by SAS, we'd have two congruent triangles: $\triangle WSI \cong \triangle MSI$.

But that would mean, by CPCTC, that $\angle SIM \cong \angle SIW$. Since we can assume from the diagram that $\angle WIM$ is a straight angle, that means $\angle SIM$ & $\angle SIW$ are supplementary. And if two angles are congruent *and* supplementary to each other (which means they add up to 180°), then they *must* both equal 90°, right? After all, if $x + x =$ 180°, then it must be true that $x = 90°$.

But wait, if $\angle SIW = 90°$, that contradicts the Given that says $\angle SIW$ is obtuse!

Since we got a contradiction, we know our assumption $\overline{SW} \cong \overline{SM}$ must have been false, and there's only one other option; it must be true that $\overline{SW} \ncong \overline{SM}$. Done!

$\therefore \overline{\boldsymbol{SW}} \ncong \overline{\boldsymbol{SM}}$

By the way, don't worry if you wouldn't have thought of this on your own. These proofs take some getting used to, and the more you do 'em, the more easily you'll come up with these strategies on your own – promise!

And here's that same proof, in two-column format!

♥Proof♥

Statements	**Reasons**
1. $\overrightarrow{SI}$ bisects $\angle WSM$	Given
2. $\angle SIW$ is obtuse	Given
3. Assume $\overline{SW} \cong \overline{SM}$	Assumption leading to a possible contradiction (Gimmie an "S"!)
4. $\angle WSI \cong \angle MSI$	Definition of bisect (see #1) (Gimmie an "A"!)
5. $\overline{SI} \cong \overline{SI}$	Reflexive Property (Gimmie an "S"!)[1]
6. $\triangle WSI \cong \triangle MSI$	SAS (3, 4, 5)
7. $\angle SIM \cong \angle SIW$	CPCTC
8. $\angle SIM$ & $\angle SIW$ are supp.	If two angles form a straight angle (assumed from diagram), then they are supplementary to each other.
9. $\angle SIM$ & $\angle SIW$ both equal 90°	If two angles are congruent *and* supplementary then they must equal 90°. (This is because if $x + x = 180°$, then it must be true that $x = 90°$) Contradicts #2; obtuse angles are strictly greater than 90°; they can't *equal* 90°.
10. $\therefore \overline{SW} \not\cong \overline{SM}$	The assumption (#3) must have been false, since Statements #2 & #9 contradict each other.

[1] Instead of using the Reflexive property, we also could have used $\overline{SW} \cong \overline{SM}$ to show that the big triangle must be isosceles, and then we'd have $\angle M \cong \angle W$, which we could use for ASA, instead of SAS. Both methods would lead to the same #6: $\triangle WSI \cong \triangle MSI$. And the proofs would be identical after that point.