Girls Get Curves

Solution Guide for Chapter 11

Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

DTM from p. 184-185

2. If Leg = 3, and Hyp = 5, then this is a 3-**4**-5 triple! Oh, if they could all be this easy…

Answer: 4

3. If Leg = 3, and Leg = 5 – this might seem like a 3-4-5 triple, but remember, the hypotenuse is always the longest side, so the hypotenuse can't be 4, can it? Let's use the formula, plugging in 3 for *a* and 5 for *b*:

$a^2 + b^2 = c^2 \rightarrow 3^2 + 5^2 = c^2 \rightarrow 9 + 25 = c^2 \rightarrow 34 = c^2 \rightarrow \boldsymbol{c = \sqrt{34}}$

(We know that *c* must be positive, so we can ignore the fact that the solution "$c = -\sqrt{34}$" also satisfies this equation. Same goes for several other problems in this chapter.)

Answer: $\sqrt{34}$

4. If Leg = 7, and Hyp = 25, then this is just a 7-**24**-25 triple. Nice!

Answer: 24

5. Ok; Leg = 1, and Leg = 2. Let's use the formula, plugging in 1 for *a* and 2 for *b*:

$a^2 + b^2 = c^2$ → $1^2 + 2^2 = c^2$ → $1 + 4 = c^2$ → $5 = c^2$ → $\boldsymbol{c} = \sqrt{5}$

Answer: $\boldsymbol{\sqrt{5}}$

6. If Leg = $5\sqrt{13}$ and Hyp = $13\sqrt{13}$, this is actually a 5-**12**-13 triple, multiplied by a factor of $\sqrt{13}$, and that means the other leg is just $12\sqrt{13}$. Nice.

Answer: $\boldsymbol{12\sqrt{13}}$

7. If Leg = $\frac{8}{71}$ and Leg = $\frac{15}{71}$, then this is an 8-15-**17** triple, multiplied by a factor of $\frac{1}{71}$! So the hypotenuse must be $\frac{17}{71}$. Very tricky!

Answer: $\boldsymbol{\frac{17}{71}}$

8. If Leg = 15 and Hyp = 25, hm – they have a factor in common, **5**. Reducing the triangle's sides, we get Leg = 3 and Hyp = 5 – yep! It's a 3-4-5 triple with a factor of **5**. So that means the other leg is $4 \cdot \mathbf{5} = 20$.

Answer: 20

9. Leg = 0.005, Leg = 0.012. This looks suspiciously like a 5-12-13 triangle, doesn't it? But what is the common factor? In other words, how do we get from 5 to 0.005? We just multiply by 0.001. So that means the hypotenuse must be $13 \cdot 0.001$ = **0.013**.

Answer: 0.013

10. Leg = 45, Hyp = 51. Do these have a common factor? Yep! The "3" trick (see p. 9 in *Math Doesn't Suck*) tells us that since 5 + 1 = 6, which is divisible by 3, that means 51 is divisible by 3! In fact, $51 = \mathbf{3} \cdot 17$. And $45 = \mathbf{3} \cdot 15$, so we have an **8**-15-17 triangle in disguise – multiplied by a factor of 3. That means the shorter leg is just $8 \cdot 3 = 24$.

Answer: 24

11. Leg = $8x$, Hyp = $17x$. This is clearly an 8-**15**-17, where everyone is multiplied by a factor of x! We don't need to know what the value of x is – we know for sure that the other leg must be $15x$.

Answer: 15*x*

12. Leg = 8, Hyp = 12. These have a common factor of 4, and if we factor out 4 from each side, we get Leg = 2 and Hyp = 3. But this isn't a Pythagorean triple! So we just need to use the formula, plugging in 8 for a and 12 for c:

$a^2 + b^2 = c^2 \rightarrow 8^2 + b^2 = 12^2 \rightarrow 64 + b^2 = 144 \rightarrow b^2 = 80 \rightarrow b = \sqrt{80}$

Now we need to reduce the radical (see p. 283-284 in *Hot X: Algebra Exposed* to review reducing radicals).

$$b = \sqrt{80} = \sqrt{16 \cdot 5} = 4\sqrt{5}$$

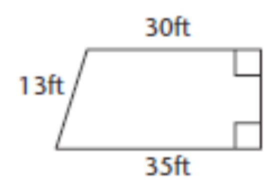
Answer: $\boldsymbol{4\sqrt{5}}$

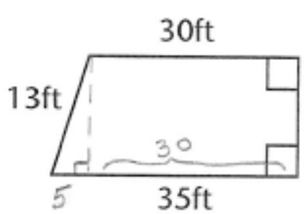
13. Leg = 45π, Hyp = 51π

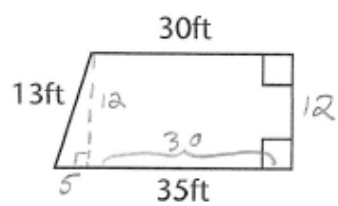
In #10, we saw that the common factor for 45 & 51 was 3. In this case, the common factor is 3π! Yep, factor out 3π from the two sides of this triangle, and we get Leg = 15, Hyp = 17. And that's an **8**-15-17 triangle with a common factor of 3π, which means the length of the shorter leg is just $8 \cdot 3\pi = 24\pi$.

Answer: 24π

14. The missing side of this trapezoid is the same as the height, and if we draw in an altitude from the upper left vertex, it must have the same length as the height! Doing that (see below), we've created a right triangle and a rectangle. The rectangle's top and bottom sides must be equal, which means the "leftover" segment on the bottom must equal 5 ft. And that's a *leg* of our right triangle.





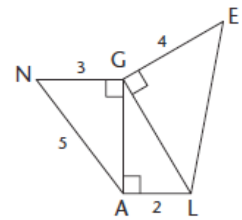


Now we have two lengths on the right triangle, a leg = 5 and hypotenuse = 13. Well that's a 5-**12**-13 triple! That means the altitude we drew has a length of 12 ft, which means the missing side of the trapezoid is also **12 ft**.

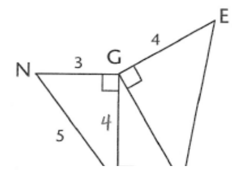
The total perimeter? We add up the outside lengths! 13 + 30 + 12 + 35 = **90**.

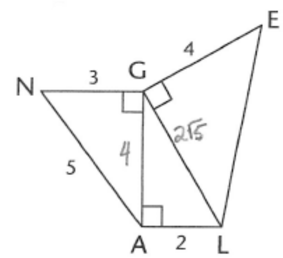
Answer: 12 ft; perimeter = 90 ft

15. We want to find EL, but first let's see what we can find out right away. The only right triangle we already have 2 sides for, is the one on the left; Leg = 3 and Hyp = 5. Great! It's a 3-**4**-5 triple, and that means $GA = 4$.



Now let's write in the "4" and we can look at the middle triangle. We have Leg = 2 and Leg = 4. Reduced by a factor of 2, that's Leg = 1 and Leg = 2 – nope, not a family, but a really easy triangle to use the Pythagorean formula to! We'll use $a = 2$ and $b = 4$, and we get:





$a^2 + b^2 = c^2 \rightarrow 2^2 + 4^2 = c^2 \rightarrow 4 + 16 = c^2 \rightarrow 20 = c^2 \rightarrow$

$c = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$

So now we know that $GL = 2\sqrt{5}$.

Writing that on the diagram, finally, we can turn our attention to the triangle on the right! For this right triangle, we have Leg = $2\sqrt{5}$ and Leg = 4. And EL, the length we want to find, is the hypotenuse! Let's use the formula with $a = 2\sqrt{5}$ and $b = 4$, and we get:

$a^2 + b^2 = c^2 \rightarrow (2\sqrt{5})^2 + 4^2 = c^2 \rightarrow (2^2 \cdot \sqrt{5}^2) + 16 = c^2 \rightarrow (4 \cdot 5) + 16 = c^2$

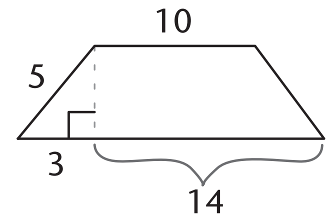
$\rightarrow 20 + 16 = c^2 \rightarrow 36 = c^2$

$c =$ **6**

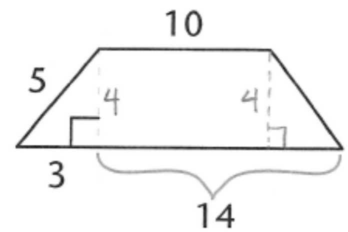
And that's the value of EL. Phew!

Answer: 6

16. We want to find the missing side and also the perimeter. Taking the hint, the first we'll draw in the missing altitude on the right side of this diagram.

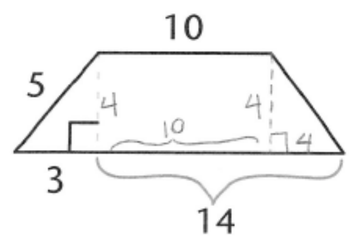


Since this is a *trapezoid* (the top and bottom segments are parallel), we've created a rectangle on the inside, and two right triangles on either side. Hey! The triangle on the left is a 3-**4**-5 triangle, which means that left-hand altitude is 4, and because the rectangle's left and right sides will always be congruent, that means the other altitude is also 4. So we can fill that stuff in right away.



Hm, how do we get that missing side on the right? Well, the rectangle's top and bottom sides must also be congruent, so that means the "leftover" segment on the bottom right must equal 14 – 10 = **4**.

Great! Now we have a right triangle on the right side with two sides: Leg = 4 and Leg = 4. And that's just half a square whose diagonal is $4\sqrt{2}$. (We also could have used the Pythagorean formula on it.)



So the missing side = $\mathbf{4\sqrt{2}}$, and the entire perimeter is all the outside lengths added up: $5 + 10 + 4\sqrt{2} + 14 + 3 =$ $\mathbf{32 + 4\sqrt{2}}$

Answer: $\mathbf{4\sqrt{2}}$ **; perimeter = 32 +** $\mathbf{4\sqrt{2}}$

17. Without even looking at a triangle, if all we're told is that the sides have lengths 21, 72 and 75, can we determine if it's a right triangle? Sure! We could do this:

$$21^2 + 72^2 = 75^2 \ ?$$

And if it's true, then it has to be a right triangle. But those numbers look huge! Let's look for a common factor in 21, 72, and 75. The only factors in 21 are 3 and 7.

Factoring 3 out of 21, we get **7**; factoring 3 out of 72, we get **24**, and factoring 3 out of 75, we get **25**. Hey! This is a 7-24-25 triple, with a common factor of 3.

So yes, this is totally a right triangle, and we didn't even need to use the Pythagorean formula to find out. Nice.

Answer: Yes; it's a member of the 7-24-25 family

DTM from p. 192-193

2. If the short leg of a 30°-60°-90° triangle is 7, that means the long leg is $7 \cdot \sqrt{3} = 7\sqrt{3}$ and the hypotenuse is $7 \cdot 2 = 14$.

Answer: longer leg = $7\sqrt{3}$, hypotenuse = 14

3. If the hypotenuse of a 30°-60°-90° triangle is 10, that means the *short* leg is 20 divided by 2, in other words, **10**, and the longer leg is the shorter leg times $\sqrt{3}$, in other words, $10\sqrt{3}$.

Answer: shorter leg = 10, longer leg = $10\sqrt{3}$

4. If the hypotenuse of a 30°-60°-90° triangle is 5, that means the short leg is 5 divided by 2, in other words, $\frac{5}{2}$, and then the *longer* leg is just the shorter leg times $\sqrt{3}$, in other words, $\frac{5\sqrt{3}}{2}$.

Answer: shorter leg = $\mathbf{\frac{5}{2}}$**, longer leg =** $\mathbf{\frac{5\sqrt{3}}{2}}$

5. If the longer leg of a 30°-60°-90° triangle is $3\sqrt{3}$, that means the shorter leg is $3\sqrt{3}$ divided by $\sqrt{3}$ in other words, $\frac{3\sqrt{3}}{\sqrt{3}}$, which we can simplify by canceling those $\sqrt{3}$'s, and we get **3** for the shorter leg, and the hypotenuse is always just the shorter leg times two, in other words: **6**.

Answer: shorter leg = 3, hypotenuse = 6

6. If the *longer* leg of a 30°-60°-90° triangle is 18, that means the shorter leg is 18 divided by $\sqrt{3}$ in other words, $\frac{18}{\sqrt{3}}$. But we should rationalize the denominator, so we'll multiply this times the copycat fraction $\frac{\sqrt{3}}{\sqrt{3}}$, and we get: $\frac{18}{\sqrt{3}} = \frac{18}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{18\sqrt{3}}{3}$ $= \mathbf{6\sqrt{3}}$. That's the *shorter* leg. And the hypotenuse is always just the shorter leg times two, in other words, $\mathbf{12\sqrt{3}}$. Done! *(If we'd multiplied the <u>unsimplified</u> shorter leg,* $\frac{18}{\sqrt{3}}$*,*

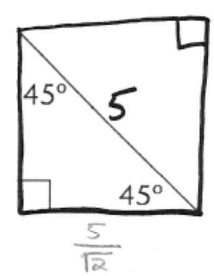
by two, we'd have gotten $\frac{36}{\sqrt{3}}$*. And then we could rationalize the denominator to get*

$12\sqrt{3}$ *. Extra work, though!)*

Answer: shorter leg = $\mathbf{6\sqrt{3}}$ **, hypotenuse =** $\mathbf{12\sqrt{3}}$

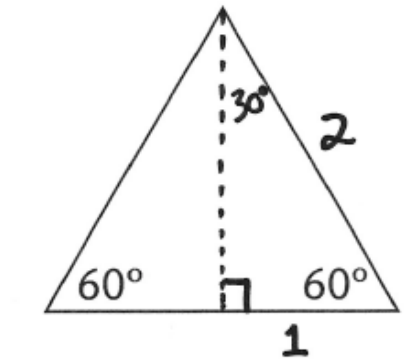
7. If the diagonal of a square is 5 inches, what is the perimeter? Let's start by finding one of the *sides* of the square, and then we'll look at one of the 45°-45°-90° triangles inside the square. Since the legs of 45°-45°-90° triangles are found by dividing the hypotenuse by $\sqrt{2}$, that means a side of this square = $\frac{5}{\sqrt{2}}$. With me so far?

Now, we'll rationalize the denominator by multiplying this times the copycat fraction $\frac{\sqrt{2}}{\sqrt{2}}$, and we get: $\frac{5}{\sqrt{2}} = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$. The perimeter will be 4 of these added together, in other words, the perimeter = $\frac{5\sqrt{2}}{2} \cdot 4 = 10\sqrt{2}$.



Answer: perimeter = $\mathbf{10\sqrt{2}}$ **inches**

8. If the perimeter of an equilateral triangle is 6 feet, and there are 3 sides, that means each side is 2 feet, right? Drawing in an altitude, we've created a 30°-60°-90° triangle with a hypotenuse of 2 and shorter side of 1.



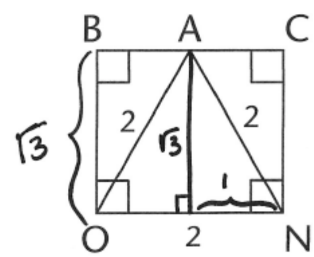
What's the altitude? It's the shorter side times $\sqrt{3}$, in other words: $\sqrt{3} \cdot 1 = \sqrt{3}$ feet.

Answer: $\sqrt{3}$ feet

9. . If the shorter leg of a 30°-60°-90° triangle is x, that means the long leg is $x\sqrt{3}$ and the hypotenuse is $x \cdot 2 = 2x$. Remember, x is just a number whose value we don't happen to know yet!

Answer: longer leg = $x\sqrt{3}$, hypotenuse = 2x

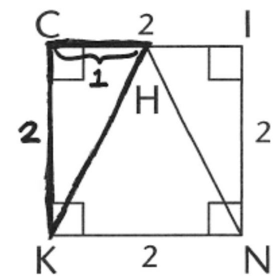
10. Drawing in the altitude from A, since we've been told that $\triangle OAN$ is an equilateral triangle, we know we've created two 30°-60°-90° triangles, and they both have a hypotenuse of 2. Looking at one of them, this means the



shorter leg must be 1, right? And that means and the longer leg must be $\sqrt{3} \cdot 1 = \mathbf{\sqrt{3}}$. Since the longer leg of our right triangle is the same length as the side of the rectangle, that means $BO = \mathbf{\sqrt{3}}$. (Make sure you followed that!) And since another side of the rectangle OBCN is 2, that means it's *not* a square!

Answer: $BO = \sqrt{3}$; No, this rectangle is not a square

11. If the square's sides are 2 inches each, and if the triangle is isosceles, that means *H* divides $\overline{CI}$ into two segments of equal length: *CH* = *HI*. And that means *CH* = *HI* = 1. Now, $\overline{HK}$ is the hypotenuse of a right triangle with Leg = 1, Leg = 2. So we can easily use the formula to find *HK*, plugging in 1 for *a* and 2 for *b*:



$a^2 + b^2 = c^2$ → $1^2 + 2^2 = c^2$ → $1 + 4 = c^2$ → $5 = c^2$ →

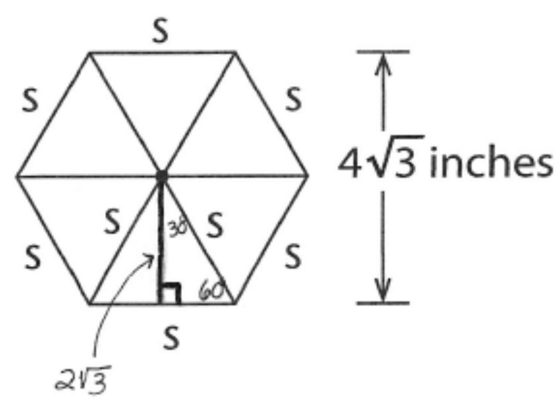
$\boldsymbol{c = \sqrt{5}}$

So $HK = \sqrt{5}$.

Since two sides of the triangle $\triangle KHN$ are 2 and one side is $\sqrt{5}$, it's definitely not equilateral!

Answer: $\boldsymbol{HK = \sqrt{5}}$; No, this triangle is not equilateral

12. If that entire height is $4\sqrt{3}$, then notice that the *altitude* of each equilateral triangle is *half* of that: $2\sqrt{3}$. Well, we can draw in one of these altitudes, and since the triangle is equilateral, it creates two a 30°-60°-90° right triangles. Looking at one of them, now we have a right triangle with a *longer* leg that measures $2\sqrt{3}$. Do you see why?



And this is great! Since the longer leg equals $2\sqrt{3}$, we know that the shorter leg must be that divided by $\sqrt{3}$, in other words, $\frac{2\sqrt{3}}{\sqrt{3}}$ = **2**, and then the hypotenuse is just twice the shorter leg, in other words, 4. Make sense so far?

And lookie there, the hypotenuse is the same as each "*s*"! So *s* = 4. And since the perimeter is just 6 of these 4's added together, that gives us 24 for the perimeter.

Answer: *s* = 4 inches; perimeter = 24 inches