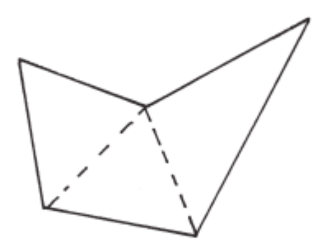
Girls Get Curves

Solution Guide for Chapter 12

Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

DTM from p. 210-212

2. Here's an example – we just draw 5 points as vertices, draw (non-intersecting) lines between them like this, and put dotted lines in for all the diagonals, and we can see how yep, 3 triangles are still formed! (Drawings will vary.)



3. If a polygon has 10 sides, then *from one vertex*, we can draw diagonals to all vertices except itself and its two "neighbors," so that's **7** diagonals, and that would create **8** triangles (it's always one more triangle than diagonal, when drawing diagonals from a single vertex – try for yourself!).

And how many *total* diagonals can be drawn in a 10-gon from all the vertices? That's our formula: $\frac{n(n-3)}{2} \rightarrow \frac{10(10-3)}{2} = \frac{10(7)}{2} = 5(7) = \mathbf{35}$

Answer: 7; 8; 35

4. Total diagonals drawn in a 1000-gon would be: $\frac{n(n-3)}{2} \rightarrow$

$\frac{1000(1000-3)}{2} = \frac{1000(997)}{2} = \mathbf{498500}$

Answer: 498,500

5. If one interior angle of a regular polygon is 135°, we can use this formula to find the number of sides on the polygon: each int. angle = $\frac{180(n-2)°}{n}$, and solve for *n*!

$135° = \frac{180(n-2)°}{n}$

$\rightarrow$ $135n = 180(n-2)$ (we multiplied both sides by *n*, and then the denominator cancels)

$\rightarrow$ $135n = 180n - 360$

$\rightarrow$ $-45n = -360$ (we subtracted $180n$ from both sides)

$\rightarrow$ $n = \frac{-360}{-45} = 8$

So the polygon has **8 sides**! For the second part of this problem, we need to find the sum of the interior angles. We *could* use the formula… but since we already know each angle is 135° and there are 8 of them, we can just do: $135° \cdot 8 = \mathbf{1080°}$

Answer: 8 sides; 1,080°

6. If one exterior angle of a regular polygon is 12°, since we know the sum of all exterior angles of every polygon is always 360°, then we can just divide 360° by 12° and get the number of sides! 360° ÷ 12° = 30. So it has 30 sides.

Now, what is the sum of the interior angles? Well, if each exterior angle is 12°, then each interior angle must be 180° – 12° = 168°, right? And if there are 30 of them, then the sum of the interior angles must be: $168° \cdot 30 = 5040°$

Answer: 30 sides; 5,040°

7. The total number of degrees in an 11-gon is given by the formula: 180(n – 2)°, where n = 11, so that's 180(11 – 2) = 180(9) = **1620°**. In a regular 11-gon, we can find the number of degrees in each angle by just dividing by 11, and we get: 1620° ÷ 11 = $147.\overline{27}°$ or in fraction form, $147\frac{3}{11}°$.

Answer: 1,620°; $\mathbf{147\frac{3}{11}°}$

8. If the sum of the interior angles in a polygon is 8,640°, then we can use this formula (and solve for n) to find the number of sides is has: Sum of int. angles = 180° (n – 2). Let's do it!

$$8640° = 180°(n - 2)$$

$$\rightarrow \frac{8640°}{180°} = n - 2$$

$$\rightarrow 48 = n - 2$$

$$\rightarrow n = 50$$

So it has 50 sides! (Note that it doesn't have to be a regular 50-gon – any 50-gon will have a sum of its exterior angles equal 8640°. Picture the (n – 2) triangles we draw inside – that will help you remember why this is true!)

Answer: 50 sides

9. The sum of exterior angles for ANY polygon is always 360°. ☺

Answer: 360°

10. The formula for the number of diagonals for an n-gon is:

$$\text{total \# of diagonals} = \frac{n(n-3)}{2}$$

So if a polygon has a total 14 diagonals, we can find the number of sides like this, solving for n:

$14 = \frac{n(n-3)}{2}$

→ $28 = n(n-3)$

→ $28 = n^2 - 3n$

→ $n^2 - 3n - 28 = 0$ (We subtracted 28 from both sides and then flipped the equation)

→ $(n + 4)(n - 7) = 0$ (See Ch. 26 in *Hot X: Algebra Exposed* for factoring equations!)

→ $n = -4, 7$

Since n can't be negative, the answer is 7. We've learned if a polygon has a total of 14 diagonals, it must have 7 sides. In other words, it's a **heptagon or 7-gon**.

Instead, if a polygon has a total of 20 diagonals, we'd do the same exact thing with 20 instead of 14:

$$20 = \frac{n(n-3)}{2}$$

$$\rightarrow 40 = n(n-3)$$

$$\rightarrow 40 = n^2 - 3n$$

$$\rightarrow n^2 - 3n - 40 = 0$$

$$\rightarrow (n+5)(n-8) = 0$$

$$\rightarrow n = -5, 8$$

Since n can't be negative, the answer is 8. We've learned if a polygon has a total of 20 diagonals, it must have 8 sides. In other words, it's an **octagon or 8-gon**.

And if a polygon has a total 35 diagonals, we can find the number of sides like this, solving for n:

$$35 = \frac{n(n-3)}{2}$$

$$\rightarrow 70 = n(n-3)$$

$$\rightarrow 70 = n^2 - 3n$$

$$\rightarrow n^2 - 3n - 70 = 0$$

$$\rightarrow (n+7)(n-10) = 0$$

$$\rightarrow n = -7, 10$$

Since n can't be negative, the answer is 10. We've learned if a polygon has a total of 35 diagonals, it must have 10 sides. In other words, it's a **decagon or 10-gon**.

Phew, done!

Answer: heptagon (7-gon); octagon (8-gon); decagon (10-gon)

11. In a regular n-gon, since the measure of each single exterior angle is $\frac{360°}{n}$, and since it must be supplementary to each interior angle, then we can express each interior angle of a regular n-gon like this: $\mathbf{180° - \frac{360°}{n}}$.

For part b, we'll simplify this answer into a single fraction by using a common denominator "n", and we'll do that by multiplying the 180 by the copycat fraction $\frac{n}{n}$ (we assume n doesn't equal 0, of course!), and simply subtracting across the top. We get:

$$180° - \frac{360°}{n} = (180°)\frac{n}{n} - \frac{360°}{n} = \frac{180n°}{n} - \frac{360°}{n} = \frac{180n° - 360°}{n}$$

This is totally equal to what we started with; it's just written differently! And now we can simplify the numerator by factoring out the common factor, 180, and we get:

$$\frac{180(n-2)°}{n}$$

And hey, that's the same formula as on p. 208 – for the measure of a single interior angle in an n-gon. Nice.

(Check out Ch. 3 in *Hot X: Algebra Exposed* to review factoring with variables – and pulling out of uncool parties)

Answer:

11a. $\mathbf{180° - \frac{360°}{n}}$

11b. $\mathbf{\frac{180(n-2)°}{n}}$

12. If there are 20 people, and everybody hugs each other, how many hugs are there? We can imagine a 20-gon, where a person stands at each vertex (like on p.209, but with 20 sides), and then each diagonal represents a hug. We can't just blindly use the formula for diagonals, though, because with people, we *can* hug our neighbors (unlike diagonals, which never connect two neighbors).

So for each person, they can hug 19 people, right? That's 19 "diagonals" we draw from that one person. We can do that 20 times, which is 19(20) diagonals, except that we've double counted, because me hugging you is the same "hug" as you hugging me! So we need to divide this by 2, which is: $\frac{19(20)}{2}$ = 190. That's a lot of hugging. ☺

Answer: 190 hugs