

Solution Guide for Chapter 14

Here are the solutions for the "Doing the Math" exercises in Girls Get Curves!

DTM from pp. 234-235

2. In the FASHN diagram, if $\textcircled{O}^\circ = 40^\circ$, can we prove that any line pairs are parallel? If so, which lines? Hm, let's tilt our heads to the right and notice that \overline{FN} could be an escalator, crossing two floors ($\overrightarrow{AH} & \overrightarrow{SN}$) at the mall! Now it becomes more clear that the angles marked by \textcircled{O}° and $\angle N$ are corresponding angles. The diagram tells us that $\angle N$ = 40°, and since we are now told that $\textcircled{O}^\circ = 40^\circ$, we know that $\angle N$ and \textcircled{O}° are congruent! And the "If corr \angle 's are \cong , then lines are ||" theorem tells us that those mall floors, \overrightarrow{AH} and \overrightarrow{SN} , must be parallel. Nice!

Answer: Yes, $\overrightarrow{AH} \parallel \overrightarrow{SN}$

3. In the FASHN diagram, if $\angle S = 40^\circ$, can we prove that any line pairs are parallel? If so, which lines? Can we prove anything else about this diagram?

Looking carefully at the diagram, we notice that the only two lines that don't intersect, and therefore, the only two lines that could *possibly* be parallel, are \overrightarrow{AH} and \overrightarrow{SN} , right? (Take a look and see



how that's true.) So that means the only possible escalators could be \overrightarrow{FS} or \overrightarrow{FN} . But $\angle N$ doesn't touch the \overrightarrow{FS} escalator. And $\angle S$ doesn't touch the \overrightarrow{FN} escalator. Notice that all of our theorems depend on *a single escalator that touches the two angles in question*. So we can't conclude that \overrightarrow{AH} and \overrightarrow{SN} are parallel. We just don't have the right information.

Can we conclude anything else about the diagram? Hm, if we know that $\angle S = 40^\circ$, then looking at the big triangle $\triangle FSN$, we can see that since $\angle S = 40^\circ$, and $\angle N = 40^\circ$, it must be true that $\angle F = 100^\circ$, since the angles in every triangle add up to 180°. Oh, and hey, if two angles in a triangle are equal, then it's an isosceles triangle, isn't it? **Answer: We cannot prove any lines are parallel.**

And yes, $\triangle FSN$ is isosceles, and $\angle F = 100^{\circ}$.

4. In the LOVED diagram: Given $\angle 1 \cong \angle 2$. Which pair of lines is parallel? Which Rule guarantees that?



Let's imagine extending the lines \overrightarrow{LO} and \overrightarrow{VE} , and then tilt our heads to the left. Do you



see how we've created a mall with two floors, and now the long line \overrightarrow{LD} could be an escalator, because it crosses both "floors"? And notice that $\angle 1 \& \angle 2$ are corresponding angles. Since we're given

 $\angle 1 \cong \angle 2$, that means the mall floors, \overrightarrow{LO} and \overrightarrow{VE} , must be parallel! In other words: $\overrightarrow{LO} \parallel \overrightarrow{VE}$. The Rule that we just used is: "If corr $\angle s$ are \cong , then lines are \parallel ." Done! Answer: $\overrightarrow{LO} \parallel \overrightarrow{VE}$; If corr $\angle s$ are \cong , then lines are \parallel ."

5. In the LOVED diagram, Given: $\angle 1 \cong \angle 2$ and $\angle O \& \angle E$ are right angles. Can we figure out $\angle OVE$? If so, what is it?

Since we have the same Given as in #4, $\angle 1 \cong \angle 2$, we can use our answer from #4 to help us. We learned that $\overrightarrow{LO} || \overrightarrow{VE}$. So we have two floors at the mall! Let's actually extend the lines and tilt the diagram, and then we can see the big Z. Looking at the Z, we see that $\angle O \& \angle OVE$ are *alternate interior angles*, which means they MUST be equal, since $\overrightarrow{LO} || \overrightarrow{VE}$!

So, since $\angle O = 90^\circ$, we also know that $\angle OVE = 90^\circ$.

Answer: Yes, $\angle OVE = 90^{\circ}$



6. In SINGER, Given: $\angle 3$ is supplementary to $\angle 4$.

Prove: $\overrightarrow{SN} \parallel \overrightarrow{IG}$, using a two-column proof.

Hm, if we tilt our heads to the left, we can see the mall floors



 $\overline{SN} \& \overline{IG}$ (the two lines we want to prove are parallel), with the escalator \overline{SR} . We are told that $\angle 3$ and $\angle 4$ are supplementary, and looking at our mall and transversal (escalator) \overline{SR} , we can see these angles are *same-side interior angles* – so the "If same side int. \angle 's are supp, then lines are \parallel " Rule tells us that the mall floors must be parallel! In other words: $\overline{SN} \parallel \overline{IG}$. Nice.

<u>Statements</u>	Reasons
1. $\angle 3$ is supp to $\angle 4$	1. Given
2. $\overrightarrow{SN} \parallel \overrightarrow{IG}$	2. If same side int. \angle 's are supp, then lines are \parallel

7. As shown in SINGR, the lines \overrightarrow{SI} & \overrightarrow{NG} intersect. Prove $\angle ISN$ is not supplementary to $\angle SNG$ with a proof by contradiction, in paragraph form.

Ah, proofs by contradiction – this is when we start out by assuming the opposite of what we're supposed to prove, and then show that it



would lead us inevitably to a false conclusion...meaning that our "assumption" had to have been false to begin with. Let's do it!

So, since we're supposed to prove that $\angle ISN$ is not supplementary to $\angle SNG$, we will assume the opposite, and see where it leads us. So let's assume that $\angle ISN$ is

supplementary to $\angle SNG$. Looking at \overline{SN} as an escalator at the mall, that would mean that $\angle ISN \ \& \angle SNG$ are same-side interior angles. And then the "If same side int. \angle 's are supp, then lines are ||" Rule tells us that the mall floors must be parallel – in other words, that $\overline{SI} || \overline{NG}$. But wait! Those two lines intersect, so they cannot be parallel. And there's a perfectly good contradiction that tells us our assumption ($\angle ISN$ is supplementary to $\angle SNG$) must have been false, which means that $\angle ISN$ is NOT supplementary to $\angle SNG$. Done!

Statements	Reasons
1. \overrightarrow{SI} and \overrightarrow{NG} intersect	1. Given
2. Assume $\angle ISN$ is supp to	2. Assumption leading to possible contradiction
∠SNG	
3. $\overrightarrow{SI} \parallel \overrightarrow{NG}$	3. If same side int. \angle 's are supp, then lines are
	. Contradicts #1, since parallel lines can never
	intersect!
4. $\therefore \angle ISN$ is not supp to	4. Our assumption (#2) must have been false.
∠SNG	

8. In STORE, Given: $\overline{ST} \cong \overline{RE}$. Prove: $\overline{SE} \not\cong \overline{TR}$ in a paragraph proof.

Let's take the hint, and draw \overline{SR} , and we'll go ahead and mark $\overline{ST} \cong \overline{RE}$ with double kitty scratches. Then, notice that if we assume that $\overline{SE} \cong \overline{TR}$ (Gimmie an "S"!), we can create a pair of congruent triangles with SSS! After all, in addition to this assumption, we are given that $\overline{ST} \cong \overline{RE}$ (Gimmie an "S"!), and the Reflexive property lets us say $\overline{SR} \cong \overline{SR}$. (Gimmie an "S"!) This gives us congruent triangles



S T R

with this correspondence of letters: $\triangle SRT \cong \triangle RSE$ (The order of these letters is very important – make sure you see why this is the order). So our assumption has led us inevitably to this point, and it continues to lead us! See, by CPCTC, we would then know that $\angle TSR \cong \angle ERS$. Let's tilt our heads to see the mall: The floors are $\overline{SO} \otimes \overline{EO}$, and the escalator is \overline{SR} , and $\angle TSR \cong \angle ERS$ means we should look at the big "Z" made up from our hand-drawn line and the two lines that have double kitty scratches (see the "Z"?). But... if $\angle TSR \cong \angle ERS$, then that means we have congruent alternate interior angles, so by the Rule "If alt. int. \angle 's are \cong , then lines are \parallel ," it would *have* to be true that $\overline{SO} \parallel \overline{EO}$! See the problem? $\overline{SO} \otimes \overline{EO}$ actually intersect on the diagram, so they cannot possibly be parallel. And this means our assumption, $\overline{ST} \cong \overline{RE}$, must have been false to begin with. And if that's a false statement, then this must be a true statement: $\overline{SE} \not\cong \overline{TR}$. Pant, pant. Ta-da!

$$\therefore SE \not\simeq TR$$

And here it is as a two-column proof, too:

<u>Statements</u>	Reasons
1. $\overline{ST} \cong \overline{RE}$	Given
2. \overline{SO} and \overline{EO} intersect	Given (visible on diagram)
3. Assume $\overline{SE} \cong \overline{TR}$	Assumption leading to possible contradiction
4. Draw \overline{SR}	Two points determine a line.
5. $\overline{SR} \cong \overline{SR}$	Reflexive Property
$6. \ \triangle SRT \cong \triangle RSE$	SSS (1, 3, 4)
7. $\angle TSR \cong \angle ERS$	СРСТС
8. $\overrightarrow{SO} \parallel \overrightarrow{EO}$	If alt. int. \angle 's are \cong , then lines are \parallel .
	Contradicts #2.
$9. \therefore \overline{SE} \not\simeq \overline{TR}$	Our assumption (#3) must have been false.

That one was pretty tricky, so don't worry if you couldn't have done it on your own. The important thing is that you read it until you understand it, and so you could explain it to someone else. Go ahead and try it!

9. Prove this Rule:

If two lines are both \perp to a third line, then they must be \parallel .

Draw a diagram with two horizontal lines, m and n, both perpendicular to a vertical (upand-down) line, l. Use a Rule from this chapter to prove that m || n. Ok, let's draw this!



In order to use any Rules from this chapter, we'll need to be using angles, right? We are given that $m \perp l$ and also $n \perp l$, so let's pick two angles and label them as $\angle 1 \& \angle 2$. We picked corresponding angles in this case (but there are other ways to do this). Because of the definition of perpendicular, we can say $\angle 1 \& \angle 2$ are both right angles, in other words, they each equal 90 degrees. That means they are congruent to each other, obviously! And since $\angle 1 \& \angle 2$ are corresponding angles, then by the Rule: "If corr. \angle 's are \cong , then lines are ||," we've proven that $m \parallel n$. Done!

By the way, we could have drawn in angles in other spots, and then we might have ended up using a different rule, like "If same side int. \angle 's are supp., then lines are \parallel ," or "If alt. ext. \angle 's are \cong , then lines are \parallel ," etc. All of the combinations of the two right angle placements (which indicate the perpendicularity stated in the Givens) totally work for this proof.

Here's what we did as a two-column proof, too!

<u>Statements</u>	Reasons
1. $m \perp l$ and $n \perp l$	1. Given
2. $\angle 1 \& \angle 2$ are right angles	2. If lines are \perp , then they create right angles
3. $\angle 1 \cong \angle 2$	3. All right angles are congruent.
4. $\angle 1 \& \angle 2$ are corresponding	4. Assumed from diagram.
angles	
$5. \therefore m \parallel n$	5. If corr. \angle 's are \cong , then lines are

Note: Statement/Reason 4 above could have been eliminated completely. In fact, any time the "Reason" is "Assumed from diagram," that step is optional, but sometimes we keep it in just to make the proof easier to follow.

Nice work in this chapter! (Hey, nobody said this stuff would be easy...)