# Girls Get Curves

## Solution Guide for Chapter 15

Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

**DTM from p. 248**

2. A trapezoid is a parallelogram.

Let's rewrite this to say: A trapezoid *satisfies the definition of* a parallelogram. Is this always, sometimes or never true? Well, parallelograms always have *two sets* of parallel sides, and trapezoids always have *exactly* one set of parallel sides (and never two!) so this statement is never true.

**Answer: Never**

3. A kite is a rectangle.

Let's rewrite this to say: A kite *satisfies the definition of* a rectangle. Is this always, sometimes or never true? Well, there are certainly kites that aren't rectangles. After all, most kites don't have four right angles, do they? However, a square satisfies the definition of "kite" and "rectangle," because every square's sides are congruent to each other (so certainly, yes, squares have "two disjoint pairs of consecutives sides congruent"), and squares also have four right angles. So a quadrilateral *can* be (satisfy the definition of) both a kite and a rectangle – when it's a square!

**Answer: Sometimes**

4. A square is a rhombus.

Let's rewrite this to say: A square *satisfies the definition of* a rhombus. Is this always, sometimes or never true? Well, a square ALWAYS satisfies the definition of "rhombus," because a square will always have two sets of parallel sides and two sets of opposite congruent sides, right? So the answer is **always**. Plus, we can see from the flowchart on p. 245 that the square is actually a "descendant" of the rhombus – so that means it automatically satisfies the definition of rhombus. ☺

**Answer: Always**

5. A trapezoid is an isosceles trapezoid.

Let's rewrite this to say: A trapezoid *satisfies the definition of* an isosceles trapezoid. Is this always, sometimes or never true? Well, it's sometimes true, right? I mean, a trapezoid satisfies the definition of isosceles trapezoid *sometimes*… when it's an isosceles trapezoid!

**Answer: Sometimes**

6. A parallelogram is a rectangle.

Let's rewrite this to say: A parallelogram *satisfies the definition of* a rectangle. Is this always, sometimes or never true? Well, this is sometimes true, because if a parallelogram happens to have 4 right angles, then will, by definition, be a rectangle! But most parallelograms don't satisfy the definition of rectangles – like any parallelogram that doesn't have right angles!

**Answer: Sometimes**

7. An isosceles trapezoid is a trapezoid.

Let's rewrite this to say: An isosceles trapezoid *satisfies the definition of* a trapezoid. Is this always, sometimes or never true? This is always true! An isosceles trapezoid is just a type of trapezoid, so it always satisfies its definition.

**Answer: Always**

8. A rhombus is a kite.

Let's rewrite this to say: A rhombus satisfies *the definition of* a kite. Is this always, sometimes or never true? Well, rhombuses always have 4 congruent sides, so that means the definition of kite, "a quadrilateral with two sets of disjoint consecutive congruent sides," is *always* satisfied.

**Answer: Always**

9. A square is an isosceles trapezoid.

Let's rewrite this to say: A square *satisfies the definition of* an isosceles trapezoid. Is this always, sometimes or never true? Well, a square is super-satisfying after all… but the trapezoid is the one type of shape a square does *not* satisfy! That's because trapezoids are only allowed to have *one* set of parallel sides, and squares always have *two* sets.

**Answer: Never**

10. A rhombus is a square.

Let's rewrite this to say: A rhombus *satisfies the definition of* a square. Is this always, sometimes or never true? This is sometimes true – when rhombuses have 4 right angles! But there are certainly rhombuses that don't have 4 right angles, so it's not *always* true.

**Answer: Sometimes**

11. Name a shape that is a parallelogram but that isn't always a square or rhombus.

Hm, looking at the chart on p. 245, **rectangles** are always parallelograms, but since rectangles don't always have 4 congruent sides, they're *not* always squares or rhombuses. We have found one!

**Answer: rectangle (of course, "parallelogram" could be an answer, too!)**

12. Name a shape that is both a kite and a rectangle.

Ah, the super-satisfying square strikes again! The **square** is the only shape that always satisfies the definitions of both kites and rectangles.

**Answer: square**

13. Which shape is equilateral but not equiangular?

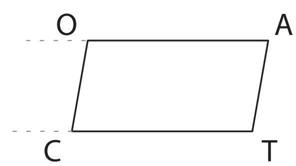
Hm, a quadrilateral with 4 congruent sides but not 4 congruent angles? That would be the rhombus!

**Answer: rhombus (but a rhombus that isn't a square, of course!)**

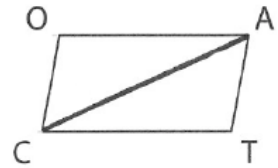
**DTM from p. 255**

2. In the diagram, $\square COAT$ is a parallelogram.

Prove that $\angle O \cong \angle T$ .



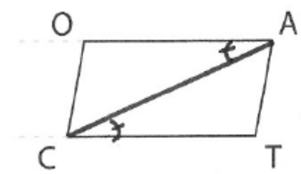

Ok, so we want to prove that those two opposite angles are congruent: $\angle O \cong \angle T$ . How can we do that? Let's start by creating some triangles – and follow the golden rule on p. 241 by drawing in a diagonal! We'll draw in the diagonal that does NOT cross through $\angle O$ & $\angle T$ , which makes the most sense, right? That way we'll end up with triangles that have $\angle O$ & $\angle T$ as angles in them.



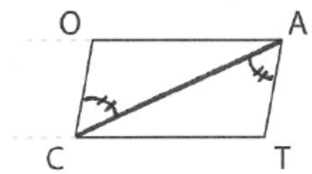

Okay, now that we've drawn in our diagonal, we see two triangles. Might they be congruent? If they were, with the point

$O$ corresponding to the point $T$, then we could use CPCTC to prove that $\angle O \cong \angle T$, right? Sounds like a plan! Time to start collecting A's and S's…

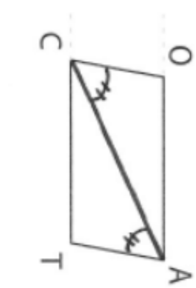
Right off the bat, we know we have one "S" – the Reflexive Property tells us that $\overline{CA} \cong \overline{CA}$. But now what? Well, we also should remember that we've been given a parallelogram, so we know that both sets of opposite sides are *parallel*, and that gives us two malls & escalators with parallel floors – which can definitely lead to congruent angles.




For instance, since $\overline{OA} \parallel \overline{CT}$ (and pretending that $\overline{CO}$ & $\overline{TA}$ don't exist), the big "Z" tells us that $\angle OAC \cong \angle TCA$ - Gimmie an "A"!




Then, turning our heads sideways, since $\overline{CO} \parallel \overline{TA}$ (and pretending that $\overline{OA}$ & $\overline{CT}$ don't exist – cover them up with your finger!), the big sideways "N" tells us that $\angle OCA \cong \angle TAC$. Gimmie an "A"!



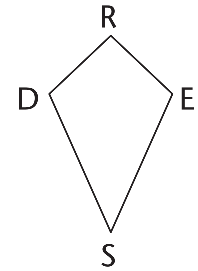
*Or, tilting our heads, we can see the backwards Z – use whichever one is easier for you to "see"! Remember – the angles always must touch the parallel mall floors that we're currently using, and also must touch the escalator. It can be tricky, but just pay attention and you'll do great!*

And now, by ASA, we've proven that $\triangle OAC \cong \triangle TCA$, which is great, since CPCTC now tells us that all corresponding triangle parts are congruent, including what we wanted to prove: $\angle O \cong \angle T$. Now we're ready to put it in two-column form:

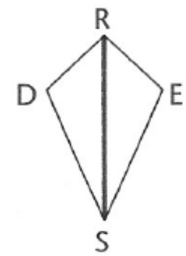
| **Statements** | **Reasons** |
|---|---|
| 1. $\square COAT$ is a $\parallel$-ogram | 1. Given |
| 2. Draw $\overline{CA}$ | 2. Two points determine a line. |
| 3. $\overline{OA} \parallel \overline{CT}$ and $\overline{CO} \parallel \overline{TA}$ | 3. If a quad is a $\parallel$-ogram, then its opposite sides are $\parallel$. |
| 4. $\angle OAC \cong \angle TCA$ | 4. If $\parallel$, then alt int $\angle$'s are $\cong$. (Gimmie an "A"!) |
| 5. $\angle OCA \cong \angle TAC$ | 5. If $\parallel$, then alt int $\angle$'s are $\cong$. (Gimmie an "A"!) |
| 6. $\overline{CA} \cong \overline{CA}$ | 6. Reflexive Property (Gimmie an "S"!) |
| 7. $\triangle OAC \cong \triangle TCA$ | 7. ASA (4, 6, 5) |
| 8. $\therefore \angle O \cong \angle T$ | 8. CPCTC |

By the way, there many ways to do that proof – particularly if you have already been introduced to a theorem in your class that says something to the effect of, "If two angles are supplementary to the same angle, then the two angles are congruent." In that case, we could actually avoid the triangle/CPCTC method completely! We could first use the "If $\parallel$, then same side int. $\angle$'s are supp" Rule to prove $\angle O$ is supp to $\angle C$ and then apply that same Rule again to prove that $\angle T$ is supp to $\angle C$. Then using the "If two angles are supplementary to the same angle, then the two angles are congruent" Rule, we could prove that $\angle O \cong \angle T$. There are usually a few ways to do these proofs – and it's great practice to try different ways (if you have the time!).

3. Given: DRES is a kite with $\overline{DR} \cong \overline{RE}$. Prove that the side angles are congruent: $\angle D \cong \angle E$.




Ah, the golden rule shall save us again! Let's draw in a diagonal – which one? Well, again, let's draw in the one that doesn't cut up the two angles we are trying to prove are congruent. So that means we'll draw in $\overline{RS}$.




Now gosh, if we could only prove that $\triangle RDS \cong \triangle RES$ (and they sure seem to be), then CPCTC will tell us that $\angle D \cong \angle E$. Very similar strategy as in #2. So how do we get our congruent triangles this time?

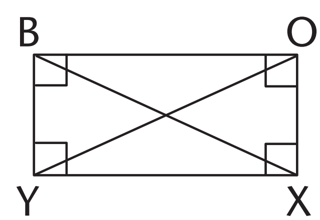
We are actually given that $\overline{DR} \cong \overline{RE}$, so there's one "S". And again, the Reflexive property will give us an "S." Now what? Well a Given that we haven't used yet is that DRES is a kite! And the definition of kite tells us that two pairs of disjoint consecutive sides are congruent. So that tells us that $\overline{DS} \cong \overline{ES}$ (and we already knew that the other pair of disjoint consecutive sides are congruent, $\overline{DR} \cong \overline{RE}$, because it was in the Given.) And now we have SSS, so we get two congruent triangles, and that's what we needed to use CPCTC! Let's write it out in two-column style:

| **Statements** | **Reasons** |
|---|---|
| 1. $DRES$ is a kite with $\overline{DR} \cong \overline{RE}$ | 1. Given (Gimmie an "S"!) |
| 2. Draw $\overline{RS}$ | 2. Two points determine a line. |
| 3. $\overline{DS} \cong \overline{ES}$ | 3. Definition of kite: if a quad is a kite, then two |

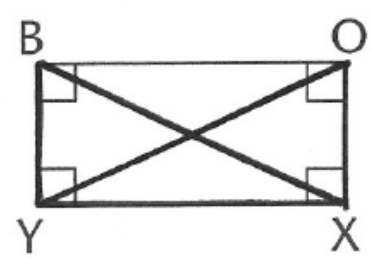
| | |
|---|---|
| | pairs of disjoint consecutive sides are $\cong$ (the other pair was our Given). (Gimmie an "S!") |
| 4. $\overline{RS} \cong \overline{RS}$ | 4. Reflexive Property (Gimmie an "S"!) |
| 5. $\triangle RDS \cong \triangle RES$ | 5. SSS (1, 3, 4) |
| 6. $\therefore \angle D \cong \angle E$ | 6. CPCTC |

Ta-da! This is a very typical style of proof to be familiar with. It's all about knowing those darn definitions… ;)

4. Given: $\square BOXY$ is a rectangle. Prove that the diagonals are congruent in a paragraph proof: $\overline{BX} \cong \overline{YO}$.




Let's see if we can use congruent triangles again – after all, if those diagonals are corresponding sides on two congruent triangles, then CPCTC will finish this off for us! Let's consider the two lower, right triangles that each have a diagonal as their hypotenuse, and try to prove that $\triangle BYX \cong \triangle OXY$. Since a rectangle is a parallelogram (satisfies the definition of!), we know that its opposite sides are congruent. That means $\overline{BY} \cong \overline{OX}$. (Gimmie an "S"!). Also, since the definition of rectangle says all its angles measure 90°, that means $\angle X \cong \angle Y$ (Gimmie an "A"!). The Reflexive property tells us that $\overline{YX} \cong \overline{YX}$, so by SAS, we've proven that $\triangle BYX \cong \triangle OXY$. Great progress!

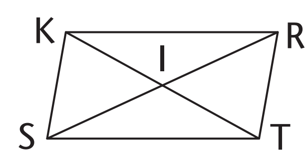



Now, because $\overline{BX}$ & $\overline{YO}$ are corresponding sides on congruent triangles, CPCTC says they must be congruent: $\overline{BX} \cong \overline{YO}$. And that's what we wanted to prove. Done!

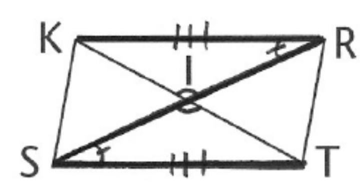
Here it is as a two-column proof, too, just in case you wanted to see it:

| **Statements** | **Reasons** |
|---|---|
| 1. $\square BOXY$ is a rectangle | 1. Given |
| 2. $\overline{BY} \cong \overline{OX}$ | 2. Opposite pairs of sides in a rect. are $\cong$. (Gimmie an "S"!) |
| 3. $\angle X$ & $\angle Y$ both = 90° | 3. In a rect, all $\angle$'s measure 90° |
| 4. $\angle X \cong \angle Y$ | 4. If two angles have the same measure, then they are $\cong$. (Gimmie an "A"!) |
| 5. $\overline{YX} \cong \overline{YX}$ | 5. Reflexive Property (Gimmie an "S!") |
| 6. $\triangle BYX \cong \triangle OXY$ | 6. SAS (2, 4, 5) |
| 7. $\therefore \overline{BX} \cong \overline{YO}$ | 7. CPCTC |

5. $\square SKRT$ is a parallelogram. Prove that $\overline{SR}$ bisects $\overline{KT}$.



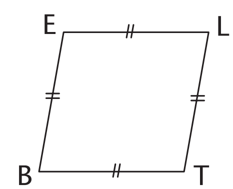

This strategy is fully explained on p. 254 – the challenge here was to write it out in two-column form, so here it is!



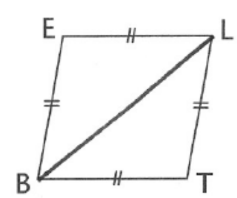

| **Statements** | **Reasons** |
|---|---|
| 1. $\square SKRT$ is a parallelogram. | 1. Given |

| | |
|---|---|
| 2. $\overline{KR} \cong \overline{ST}$ | 2. If a quad is a $\parallel$-ogram, its opposite sides are $\cong$. (Gimmie an "S"!) |
| 3. $\angle KIR \cong \angle TIS$ | 3. Vertical angles are $\cong$. (Gimmie an "A"!) |
| 4. $\overline{KR} \parallel \overline{ST}$ | 4. If a quad is a $\parallel$-ogram, its opposite sides are $\parallel$. |
| 5. $\angle KRS \cong \angle TSR$ | 5. If $\parallel$, then alt int $\angle$'s are $\cong$. (Gimmie an "A"!) |
| 6. $\triangle KIR \cong \triangle TIS$ | 6. SAA (2, 3, 5) |
| 7. $\overline{KI} \cong \overline{IT}$ | 7. CPCTC |
| 8. $\therefore$ $\overline{SR}$ bisects $\overline{KT}$ | 8. Definition of bisect |

6. In BELT, Given: $\overline{BE} \cong \overline{EL} \cong \overline{LT} \cong \overline{BT}$. Prove: $\overline{EL} \parallel \overline{BT}$. *(This is the toothpick example I mentioned in the footnote on p.242!*

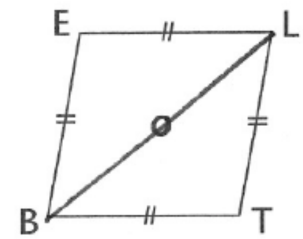



Hm, how can we prove that two lines are parallel? Well, we have a whole bunch of Rules on p. 230 that prove two lines are parallel – if we have a transversal (escalator) and enough info about the angles! We have no info about angles here, so let's start by following the golden rule and drawing in a diagonal. Which one? It doesn't matter - either will work – so let's pick $\overline{BL}$.




Great. Now we have two triangles that look like they are congruent! If we can prove they are congruent, then perhaps CPCTC will give us some angle info that we can then use on the escalator we drew and get some parallel lines. Good strategy!

So – let's try proving $\triangle BEL \cong \triangle LTB$. We already know that all of the sides are congruent from the Givens: $\overline{BE} \cong \overline{EL} \cong \overline{LT} \cong \overline{BT}$ , which certainly means that opposite sides are congruent: $\overline{BE} \cong \overline{LT}$ & $\overline{EL} \cong \overline{BT}$ and the Reflexive property tells us that $\overline{BL} \cong \overline{BL}$, so now we have SSS so we know for sure that $\triangle BEL \cong \triangle LTB$. To understand the correspondence, imagine putting a pin at the midpoint of $\overline{BL}$ and swinging the top triangle down so it lands on top of the bottom triangle – that is the congruency correspondence we are using. (*Notice that these two triangles are congruent with a different correspondence, too:* $\triangle BEL \cong \triangle BTL$*, but that won't be as helpful for what we're going to do next.*)




Awesome. Now that we have $\triangle BEL \cong \triangle LTB$, CPCTC tells us that the corresponding angles in these two triangles are congruent. But which angles would be most helpful for proving that $\overline{EL} \parallel \overline{BT}$ ? Looking $\overline{EL}$ & $\overline{BT}$ as the floors of the mall, and $\overline{BL}$ as our escalator, check out the big "Z"! If we could prove that $\angle ELB \cong \angle TBL$, then the Rule "If alt. int. $\angle$'s are $\cong$, then lines are $\parallel$" would tell us that $\overline{EL} \parallel \overline{BT}$ . And sure enough, CPCTC tells us that $\angle ELB \cong \angle TBL$. Yippee! Let's write it out:

| **Statements** | **Reasons** |
|---|---|
| 1. $\overline{BE} \cong \overline{LT}$ | 1. Given (Gimmie an "S"!) |
| 2. $\overline{EL} \cong \overline{BT}$ | 2. Given (Gimmie an "S"!) |
| 3. Draw $\overline{BL}$ | 3. Two points determine a line. |
| 4. $\overline{BL} \cong \overline{BL}$ | 4. Reflexive Property (Gimmie an "S"!) |

| | |
|---|---|
| 5. $\triangle BEL \cong \triangle LTB$ | 5. SSS (1, 2, 4) |
| 6. $\angle ELB \cong \angle TBL$ | 6. CPCTC |
| 7. $\therefore \overline{EL} \parallel \overline{BT}$ | 7. If alt. int. $\angle$'s are $\cong$, then lines are $\parallel$. |

And we could do a similar proof to show that $\overline{EB} \parallel \overline{LT}$, which means we would have proved that both sets of opposite sides are parallel. And that's the definition of rhombus! So, yep, if you put 4 toothpicks together to form a quadrilateral, it *has* to be a rhombus.

Great job!!