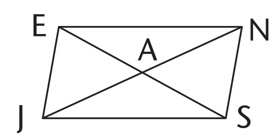
Girls Get Curves

Solution Guide for Chapter 16

Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

DTM from p. 261



2. $\overline{JA} \cong \overline{AN}$?

Property #4 on p. 260 says the diagonals of parallelograms bisect each other, which means A is the midpoint of $\overline{JN}$, so yep, $\overline{JA} \cong \overline{AN}$!

Answer: Yes, it has to be true.

3. $\overline{ES} \perp \overline{JN}$?

Do the diagonals on a parallelogram have to be perpendicular? Nope! We can definitely draw a parallelogram with diagonals that don't cross at 90°.

Answer: No, it doesn't have to be true.

4. $\angle JSE \cong \angle SEN$?

Those are angles in a big backwards "Z" at a mall with the horizontal lines as the floors, and $\overline{ES}$ as the escalator. Since this is a parallelogram, those horizontal lines have to be parallel, so the Rule, "If $\parallel$ lines, then alt. int. $\angle$'s are $\cong$" tells us that yep, $\angle JSE \cong \angle SEN$!

Answer: Yes, it has to be true.

5. $\angle JSN$ supp. to $\angle ENS$?

Yep! Imagine a mall with the horizontal lines as the floors, and $\overline{NS}$ as the escalator this time. Since this is a parallelogram, those horizontal lines *have* to be parallel, so the Rule, "If $\parallel$ lines, then same side int. $\angle$'s are supp" tells us that yep, $\angle JSN$ is supplementary to $\angle ENS$.

Answer: Yes, it has to be true.

6. $\overline{JA} \cong \overline{AS}$?

Let's not answer too quickly on this one. On the diagram, it seems like they might be congruent, but all we know about the lengths of these diagonals is that each diagonal is bisected – so we know that $\overline{JA} \cong \overline{AN}$ and also that $\overline{EA} \cong \overline{AS}$, but that's it! (As it turns out, $\overline{JA} \cong \overline{AS}$ ONLY if this parallelogram is actually a rectangle or square!)

Answer: No, it doesn't have to be true.

7. $\overline{JE} \cong \overline{SN}$?

Yes! By Property #2 on p. 259, opposite sides are congruent.

Answer: Yes, it has to be true.

8. $\overline{SA} \cong \overline{NA}$?

Nope – same reasoning as in problem #6!

Answer: No, it doesn't have to be true.

9. $\angle JEN$ supp. to $\angle JSN$?

Actually, by Property #3 on p.260, these two angles are congruent, not supplementary! (So the only way they could be supplementary is if JENS were a square, because then those angles would both be 90°!)

Answer: No, it doesn't have to be true.

10. $\angle EJS \cong \angle ENS$?

Yes! By Property #3 on p. 260, opposite angles in parallelograms are indeed congruent.

Answer: Yes, it has to be true.

11. $\angle EAJ \cong \angle NAS$?

Yes! These are vertical angles. It wouldn't matter if we had a parallelogram or not – vertical angles are ALWAYS congruent!

Answer: Yes, it has to be true.

12. $\angle EJN \cong \angle JNS$?

Yes! If we tilt our heads and imagine a mall with floors $\overline{JE}$ & $\overline{SN}$ and escalator $\overline{JN}$, then these two angles make up the big "N," and the Rule "If $\|$ lines, then alt. int. $\angle$'s are $\cong$" tells us that these two angles are indeed congruent.

Answer: Yes, it has to be true.

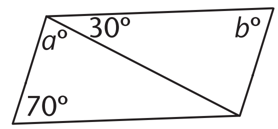
13. $\angle AJS \cong \angle ASJ$?

Nope! None of the Parallelogram Properties or our Rules say anything to indicate that this would have to be true. Similar to #12, we know that $\angle AJS \cong \angle ENJ$ and $\angle NES \cong \angle ASJ$, but not the congruency that the question is asking. In fact, the only way $\angle AJS \cong \angle ASJ$ on this diagram would be if JENS were a rectangle square, because then $\triangle JAS$ would indeed be an isosceles triangle, and those two angles would be congruent.

Answer: No, it doesn't have to be true.

DTM from p. 266

2. Given the parallelogram to the right, find $a°$ and $b°$.



Since this is a parallelogram, we know that opposite angles are congruent – so that means $\boldsymbol{b° = 70°}$! And since same-side interior angles must be supplementary, that means $a° + 30°$ is supplementary to 70° – in other words, $a° + 30° + 70° = 180°$, which we can easily solve:

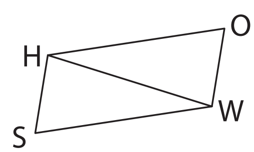
$$a° + 30° + 70° = 180°$$

$$\rightarrow a° + 100° = 180°$$

$$\rightarrow \boldsymbol{a° = 80°}$$

Answer: $\boldsymbol{a° = 80°, b° = 70°}$

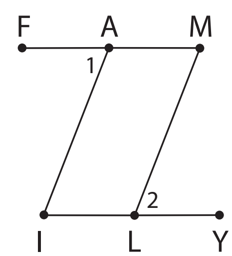
3. Given: $\triangle HSW \cong \triangle WOH$, Prove: SHOW is a parallelogram.



Since we have two congruent triangles, CPCTC tells us all sorts of information, including tons of corresponding congruent segments and angles. So there are LOTS of ways we could prove that SHOW is a parallelogram. I think the easiest way is to use the corresponding congruent segments to show that both sets of opposite sides are congruent, and then by Method #2 on p. 262, "If two pairs of opposite sides on a quad are $\cong$, then the quad is a $\|$-ogram," we will have proved that SHOW is a parallelogram. Nice!

♥Proof♥

Statements	**Reasons**
1. $\triangle HSW \cong \triangle WOH$	1. Given
2. $\overline{HO} \cong \overline{SW}$	2. CPCTC
3. $\overline{SH} \cong \overline{WO}$	3. CPCTC
4. $\therefore$ *SHOW* is a parallelogram.	4. If two pairs of opposite sides on a quad are $\cong$, then the quad is a $\parallel$-ogram.

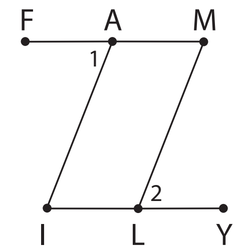


4. Given: $\overline{AM} \parallel \overline{IL}$, $\angle 1 \cong \angle 2$. Prove that *AMLI* is a parallelogram.

We have a mall with parallel floors and two escalators! And that means there will be tons of congruent angles in this diagram. Let's figure out which ones will help us prove that AMLI is a parallelogram. There are two ways that I think are easiest – one is to prove that both sets of opposite angles are congruent (so that proves AMLI is a parallelogram by Method #3 on p.262) and the other way I see is to prove that $\overline{IA} \parallel \overline{LM}$, because then we'd have both sets of opposite sides being parallel (so that proves *AMLI* is a parallelogram by Method #1 on p. 262 – this is the one I'll do below). There are probably lots of ways to do this, so don't worry if your strategy was different!

Hm, let's start by just seeing what kind of information we can figure out. We are told that $\angle 1 \cong \angle 2$, but those angles are touching different "escalators," so we can't apply any of our transversal Rules to them. See what I mean? However, since we know that

$\overline{AM} \parallel \overline{IL}$, the "If $\parallel$, then alt. int. $\angle$'s are $\cong$" Rule tells us that $\angle 2 \cong \angle M$, right? (We just used the $\overline{ML}$ escalator for the center of the big "Z"!) And then the good 'ol Transitive Property tells us that since $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle M$, that means $\boldsymbol{\angle 1 \cong \angle M}$. (Don't confuse the 1 with I – pay close attention!)



Tilting our heads sideways and looking at $\overline{AI}$ & $\overline{ML}$ as the mall floors with $\overline{FM}$ as the escalator, we can notice that $\angle 1$ & $\angle M$ are corresponding angles in this mall configuration, and since now we know $\angle 1 \cong \angle M$, that means the mall floors are parallel – in other words, the Rule "If corr. $\angle$'s are $\cong$, then $\parallel$" proves that $\overline{IA} \parallel \overline{LM}$. And since we now have two sets of opposite parallel sides, Method #1 from p. 262 is satisfied and we indeed have a parallelogram! Phew! Here it is, in two-column format:

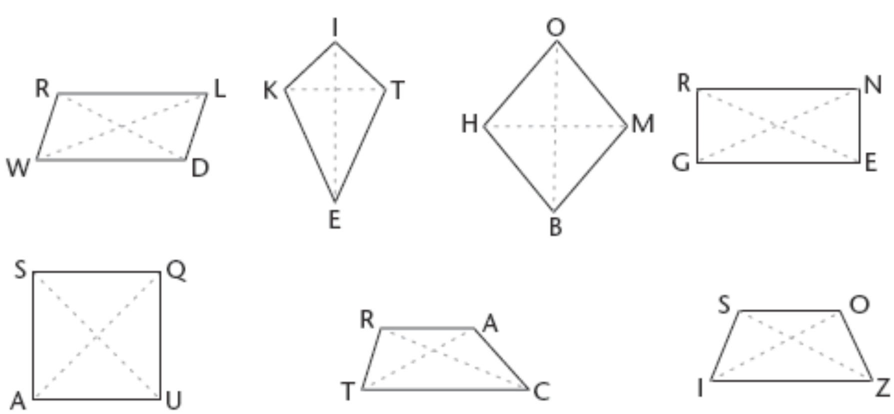
♥Proof♥

Statements	**Reasons**
1. $\overline{AM} \parallel \overline{IL}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Given
3. $\angle 2 \cong \angle M$	3. If $\parallel$, then alt. int. $\angle$'s are $\cong$.
4. $\angle 1 \cong \angle M$	4. Transitive Property
5. $\overline{IA} \parallel \overline{LM}$	5. If corr. $\angle$'s are $\cong$, then $\parallel$. *(Different mall floors & escalator this time!)*
6. $\therefore AMLI$ is a $\parallel$-ogram.	6. If two pairs opp sides of a quad are $\parallel$ (steps 1 & 5), then the quad is a $\parallel$-ogram.

If you wanted to do this proof using Method #3 on p. 262 – proving that both sets of opposite angles are congruent, here's how it might go: We could notice that the two angles supplementary to $\angle 1$ & $\angle 2$ *make up one set of opposite angles in AMLI, and since they are supplementary to congruent angles, they also must be congruent:* $\angle MAI \cong \angle MLI$.

Now for the other set of opposite angles, we could notice that since $\overline{AM} \parallel \overline{IL}$, $\angle I$ *must be congruent to* $\angle 1$ *(see the big "Z"?) and we've already seen how to prove that* $\angle 1 \cong \angle M$ *(see the above proof). The Transitive Property tells us that since* $\angle I \cong \angle 1$ *and* $\angle 1 \cong \angle M$ *, that means* $\angle M \cong \angle I$ *, and that is the other opposite pair of angles in the quadrilateral! So Method #3 on p. 262 tells us we've proven that AMLI is a parallelogram.*

DTM from p. 275-277



2. Which shape has only one pair of bisected angles? *(Who has only one bicycle that rides up hills at angles?)* Name the pair of angles.

Hm, who only has one bicycle? Can't be one of Paris' kids, because she and her descendants have two of everything. And looking at our shapes, it's the kite! The kite here (and most kites –unless they also happen to be squares) only guarantee one pair of bisected angles (see Property #4 on p.269), and those angles are the ones that the line of symmetry passes through. On the above KITE diagram, those would be $\angle I$ & $\angle E$.

Answer: kite, $\angle I$ & $\angle E$

3. For which shapes are all the angles bisected by diagonals? *(This means both diagonals are angle bisectors: Who has two bicycles that ride up angles?)*

This has got to be a child of Paris, since she has two of everything, right? And it also has to be someone who likes adventure – bicycling up angles – so that sounds like a child of Kit, too! And yep, the diagonals of a rhombus (and a square!) bisect all its angles.

Answer: rhombus, square

4. Which shapes have perpendicular diagonals? *(Who—and his or her descendants—always feels right, deep inside?)*

Ah, Kit was the self-righteous one who always felt "right" inside. And who are his descendants? Rhonda and Sarah, so that's the kite, rhombus, and square. And looking at the diagrams, yep, it makes sense!

Answer: kite, rhombus, square

5. In ISOZ, do we know that $\triangle SZI \cong \triangle OIZ$?

Yes! Here's why: ISOZ is an isosceles trapezoid, and so we know it has congruent sides ($\overline{SI} \cong \overline{OZ}$ - Gimmie an "S"!), the diagonals are congruent (see Property #4 on p. 274 – so here, $\overline{SZ} \cong \overline{OI}$ - Gimmie an "S"!), and the Reflexive property gives us the third side

($\overline{IZ} \cong \overline{IZ}$ - Gimmie an “S”!). So by SSS, $\triangle SZI \cong \triangle OIZ$.

Answer: Yes

6. In WRLD, do we know that $\triangle RWD \cong \triangle LDW$?

Hm, well we know that in a parallelogram, the opposite sides are congruent, so we’d have $\overline{RW} \cong \overline{LD}$ (Gimmie an “S”!) and the Reflexive Property tells us $\overline{WD} \cong \overline{WD}$ (Gimmie an “S”!). The diagonals of parallelograms aren’t usually congruent (not unless it also happens to be a rectangle or square), and we don’t have any other information about the sides or angles of these two triangles. In fact, these two triangles will only be congruent if WRLD is also a rectangle or square!

Answer: No

7. Name all sets of congruent diagonals. *(Imagine the full diagonals as sides of overlapping triangles.)*

Who has congruent diagonals? As we saw in #5 and #6, the diagonals being congruent led us to congruent, overlapping triangles. We know this is the case for rectangles, squares, and isosceles trapezoids. Are there any other shapes with congruent diagonals? It’s tempting to add the rhombus to the list, but we can imagine squishing the rhombus until it’s really skinny and long – just like we could with a parallelogram – and then it becomes more obvious that those don’t belong on the list! And what are the sets of congruent diagonals? Rectangle: $\overline{RE} \cong \overline{NG}$, Square: $\overline{SU} \cong \overline{QA}$, isosceles triangle: $\overline{IO} \cong \overline{SZ}$.

Answer: $\overline{RE} \cong \overline{NG}$, $\overline{SU} \cong \overline{QA}$, $\overline{IO} \cong \overline{SZ}$

8. Which shapes have two sets of opposite congruent sides? *(Who—and their kids—have two of everything?)*

This has got to be Paris! I mean, you know, the parallelogram. She has two of everything. And who are her kids? Because they have two of everything, too. In this world of quadrilaterals, your kids inherit all of their parents' traits… Her kids are Rex, Rhonda and Sarah. So that's rectangle, rhombus, and square, along with the parallelogram. Does anyone else have two sets of opposite congruent sides? Not the kite, and not the trapezoids… nope, we're done!

Answer: parallelogram, rectangle, rhombus, and square

9. Name the only shape that has sides that *don't* have to be congruent to any other sides.

The trapezoid has two sides that don't have to be congruent to any other sides!

Answer: trapezoid

10. What property is shared only by isosceles trapezoids, rectangles, and squares?

As we saw in #7, they all have congruent diagonals, and no other shapes do.

Answer: congruent diagonals

11. Which shapes also satisfy all the properties of kites and parallelograms? (Which kids are descendants of both Paris and Kit?)

Their kids are Rhonda and Sarah (well, that's a granddaughter, but it counts!) So that's the rhombus and square.

Answer: rhombus and square

12. Which shape satisfies the properties of a parallelogram, but not those of a kite (besides a parallelogram)? *(Who is Paris' kid but not Kit's kid?)*

Ah, remember, she had the love child with her European guy in that scandal – that would be Rex (the rectangle). And it's true – the rectangle doesn't satisfy the definition of kite, because it doesn't have congruent *consecutive* sides – it just has congruent *opposite* sides.

Answer: rectangle

13. What's another name for a shape that is both a rhombus and a rectangle?

If a shape has the qualities of two other shapes, it must be a descendent of them BOTH, and in this case, that would be the square!

Answer: square

14. There is one shape that doesn't have a property regarding supplementary angles. What is that shape, and why?

Well, any time we have a set of parallel lines with a transversal (escalator!), which happens for *any* quadrilateral that has a set of parallel sides, we will have the "If parallel lines, then same side int. angles are supp" Rule will apply, and guarantee supplementary angles inside the quadrilateral! But there is one shape that *doesn't* have a set of parallel lines – the kite. So kites are the only shapes that don't have a property about supplementary angles. And there you have it!

Answer: kite; it has no parallel lines

For the questions below, fill in the missing word(s).

(Hint: For #16–20, think about the "other parent" involved.)

15. Unlike the most general trapezoids, isosceles trapezoids have two _____ sides.

Isosceles trapezoids, like isosceles triangles, have two congruent sides, but the general trapezoid does not.

Answer: congruent

Unlike the most general trapezoids, isosceles trapezoids have two **congruent** sides.

16. Unlike the most general parallelograms, a rectangle's four _____are all congruent.

A rectangle doesn't have four congruent sides, but it has four congruent angles! And most parallelograms do not.

Answer: angles

Unlike the most general parallelograms, a rectangle's four **angles** are all congruent.

17. Unlike the most general kites, a rhombus' two pairs of opposite sides are ______and _____ .

A rhombus' two pairs of opposite sides are congruent and parallel – and this is not true for kites!

Answer: congruent and parallel

Unlike the most general kites, a rhombus' two pairs of opposite sides are **congruent** and **parallel.**

18. Unlike the most general parallelograms, a rectangle's diagonals _______ each other.

(Careful: Remember what we know about a parallelogram's diagonals!)

We know that's parallelograms' diagonals bisect each other, so that can't be the answer.

Hm, we know that rectangles' diagonals are congruent to each other, and that is certainly not true for most parallelograms, so that must be the answer!

Answer: are congruent to

Unlike the most general parallelograms, a rectangle's diagonals **are congruent to** each other.

19. Unlike the most general parallelograms, a rhombus' diagonals bisect _______.

Again, parallelograms' diagonals bisect each other, so that can't be the answer. But a rhombus' diagonals also bisect its angles, which is not true for most parallelograms.

Answer: their angles

Unlike the most general parallelograms, a rhombus' diagonals bisect **their angles.**

20. Unlike the most general parallelograms, a rhombus' diagonals are _____ each other.

Well, Kit (and his kids) always feel right deep inside, so yep, a rhombus' diagonals would be perpendicular to each other, just like a kite's are! And it's pretty clear that parallelograms' diagonals do not cross each other perpendicularly.

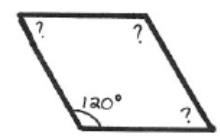
Answer: perpendicular to

Unlike the most general parallelograms, a rhombus' diagonals are **perpendicular to** each other.

DTM from p. 277-278

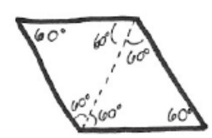
2. If one angle of a rhombus is 120°, what are its other angles? What two shapes does the shorter diagonal of this rhombus divide it into? (Draw a picture!)

Okay, let's take the hint and draw a picture. We'll draw a rhombus, which has all sides equal and both opposite pairs of sides parallel to each other. And using a protractor, we'll make sure one of the angles equals 120°. There!



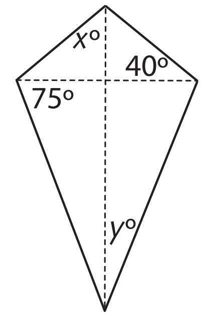
Now, what are the other angles? The opposite angle must be equal to it – so that's another 120°. And what about the smaller angles? Well, looking at the upper left angle, it must be supplementary to the angle below it (**120°**), because the top and bottom of the rhombus are parallel, and so the left side of the rhombus is like an elevator at the mall! So "If parallel, then same side int. angles are supplementary" tells us that upper left angle must be supplementary to 120°, and since 120° + **60°** = 180°, that angle must be 60°. And the lower right angle must be congruent to the upper left, since it's a rhombus (actually, all parallelograms have this quality), so we know it's **60°**, too.

For the next part, it asks us what the shorter diagonal divides the rhombus into, so let's draw that diagonal, and notice that since the diagonals of rhombuses always bisect the angles (Rhonda likes to ride her bicycles up angles!), we know that the shorter diagonal of this rhombus divides each of the 120° angles into two 60° angles, and lookie there! We can now see that the shorter diagonal actually divides this rhombus into two equilateral triangles. Nice.



Answer: 120°, 60°, 60°; two equilateral triangles

3. Given the kite to the right (with a vertical line of symmetry) find the angle measurements $x°$ and $y°$.



There are lots of ways to do this; here's one: Ok, since this is a kite, that means that two (disjoint) pairs of consecutive sides are congruent, right? In this case, since the line of symmetry is vertical, we can look on either side of the line of symmetry to see which sides must be congruent. For example, here, the top short lines are congruent – which means that top (squatty) triangle is isosceles, and that means we could write in "40°" right above the 75°. Make sense? Now, since we know the line of symmetry *bisects* the top angle, that means we could write "$x°$" to the right of the $x°$ we see on the diagram, and that top (squatty) isosceles triangle has 3 angles, measuring 40°, 40°, and $2x°$. Make sure you see why! And since all triangles' angles always add up to 180°, we can write:

$$40° + 40° + 2x° = 180°$$

$$\rightarrow 80° + 2x° = 180°$$

$$\rightarrow 2x° = 100°$$

$$\rightarrow \boldsymbol{x° = 50°}$$

Great! We can apply the same exact logic to the bottom half of this kite – the long "upside down" isosceles triangle, by filling in 75° and $y°$, and then solving the equation:

$$75° + 75° + 2y° = 180°$$

$$\rightarrow 150° + 2y° = 180°$$

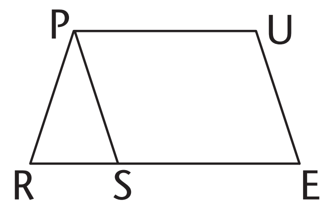
$$\rightarrow 2y° = 30°$$

$$\rightarrow \boldsymbol{y° = 15°}$$

Done!

Answer: $x°$ = 50°; $y°$ = 15°

4. Given: $\triangle RPS$ is an isosceles triangle with base $\overline{RS}$ and *RPUE* is an isosceles trapezoid. Prove: *PSEU* is a parallelogram.



We've been given a few ways to prove that a quad is a parallelogram – check out p. 262 right now to review them! You'll see that they mostly involve parallel lines, congruent sides, and the diagonals.

In this problem, we are given two sets of information involving "isosceles" shapes – which always means congruent sides and congruent angles (so we're probably *not* going to use Method #4 involving diagonals). Let's jump in!

Ok, first of all, since $\triangle RPS$ is isosceles, that means its legs are congruent: $\overline{PR} \cong \overline{PS}$. And since we know *RPUE* is an isosceles trapezoid, that means *its* legs are congruent, too: $\overline{PR} \cong \overline{UE}$. Then by the Transitive Property, we can conclude: $\overline{PS} \cong \overline{UE}$.

Nice! We've proven one pair of opposite sides of the quad PSEU are congruent. Awesome. Now all we need is either the other pair of opposite sides to be congruent for Method #2 (but we don't really have any information to lead us there…) or we could use Method #5 on p. 262, which requires one pair of opposite sides to be both congruent *and* parallel.

Hm, how can we prove that two lines are parallel? Well, with all of our "escalator" rules from p. 230, of course! So let's look for some corresponding congruent angles or big a Z or something…

Okay, if $\triangle RPS$ is an isosceles triangle, that means its base angles are congruent, right? In other words: $\angle R \cong \angle PSR$. Also, since *RPUE* is an isosceles trapezoid, that means its base angles are congruent; in other words: $\angle R \cong \angle E$. Then by the Transitive Property, we can conclude that $\angle PSR \cong \angle E$, right? And if we tilt our heads to the right, we can see a mall with $\overline{PS}$ & $\overline{UE}$ as its floors and $\overline{SE}$ as the escalator – and looking at it this way, $\angle PSR$ & $\angle E$ are corresponding angles. Can you see it? The fact that corresponding angles are congruent, $\angle PSR \cong \angle E$, means that the mall floors must be parallel! (That's one of the Rules from p. 230.) In other words, we've just proved that $\overline{PS} \parallel \overline{UE}$. Nice!

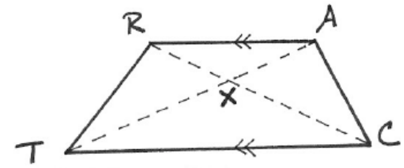
Now we have proven that one pair of opposite sides of our quad ($\overline{PS}$ & $\overline{UE}$) are parallel and congruent, and we've satisfied Method #5 from p. 262 – so we have indeed proven that *PSEU* is a parallelogram. Here is it, written out. Phew! Good job following that!

♥Proof♥

Statements	**Reasons**
1. $\triangle RPS$ is an isosceles triangle with base $\overline{RS}$	1. Given
2. $\overline{PR} \cong \overline{PS}$	2. If isosceles $\triangle$, then legs $\cong$.
3. *RPUE* is an isosceles trapezoid	3. Given
4. $\overline{PR} \cong \overline{UE}$	4. If isosceles trapezoid, legs are $\cong$.
5. $\overline{PS} \cong \overline{UE}$	5. Transitive Property *(Yay, one pair of opposite sides of PSEU is congruent!)*
6. $\angle R \cong \angle PSR$	6. If isosceles $\triangle$, then and base angles are $\cong$.
7. $\angle R \cong \angle E$	7. If isosceles trapezoid, base angles are $\cong$.
8. $\angle PSR \cong \angle E$	8. Transitive Property
9. $\overline{PS} \parallel \overline{UE}$	9. If corr. $\angle$'s are $\cong$, then $\parallel$ *(Yay, the same pair opp sides is now* $\parallel$*, too!)*
10. $\therefore$ *PSEU* is a $\parallel$-ogram.	10. If one pair of opp sides of a quad is both $\cong$ and $\parallel$ (steps 5 & 9), then the quad is a $\parallel$-ogram.

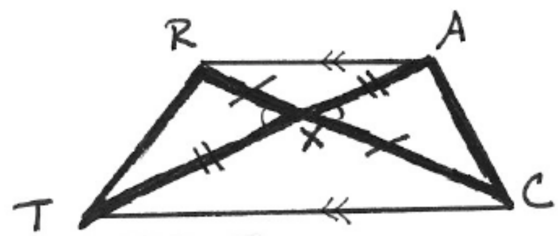
5. Explain why the diagonals of a trapezoid cannot bisect each other. *(Hint: This is an indirect proof; start by assuming they do bisect each other.)*[1]

Let's draw a trapezoid with parallel sides: $\overline{RA} \parallel \overline{TC}$.



First we'll assume the diagonals do bisect each other, and we'll try to reach a conclusion that contradicts the definition of trapezoid. Sound good? Now, the only thing that makes a quadrilateral NOT a trapezoid is if *both* sets of opposite sides are parallel. If we can do that, in other words, if we can use our "assumption" to lead us inevitably to the statement "$\overline{RT} \parallel \overline{AC}$", then we'll have gotten the contradiction we need to prove that that our assumption was nonsense in the first place! So that's our strategy. Make sense? Let's do it!

So, if we assume that the diagonals do bisect each other, that would mean $\overline{RX} \cong \overline{XC}$ and $\overline{TX} \cong \overline{XA}$, right? And if that's true, then because we have vertical angles, we can use SAS to prove the side-to-side bowtie triangles are congruent, with this correspondence: $\triangle RXT \cong \triangle CXA$. (Don't trust the lengths of the diagram as drawn – remember we're doing something "impossible" right now. To understand the correspondence, imagine one swinging triangle around the point X and lying on top of the other – T corresponds to A!)



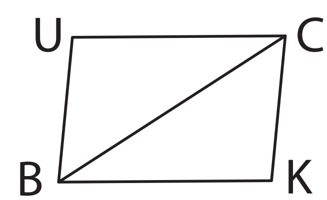
note: these drawings are based on our assumption that the diagonals bisect each other. We'll show this is impossible, if TRAC is a trapezoid!

[1] See chapter 10 to brush up on indirect proofs, AKA proofs by contradiction.

And if $\triangle RXT \cong \triangle CXA$, then CPCTC tells us $\angle RTA \cong \angle CAT$ - make sure you see which angles those are. But let's tilt our heads to the right, and see that those angles create a big backwards Z. By the Rule, "if alternate interior angles are congruent, then lines are parallel", we've proven that $\overline{RT} \parallel \overline{AC}$ - which is a direct contradiction to the definition of trapezoid! (Pant, pant!)

$\therefore$ **The diagonals of a trapezoid cannot bisect each other.**

6. Given: BUCK is a parallelogram, and $\overline{BC}$ bisects both $\angle UBK$ & $\angle UCK$. Prove: $\overline{BU} \cong \overline{UC}$. *(Hint: Remember isosceles triangles)* What else do we know about the sides? We can now prove *BUCK* satisfies the definition of a particular type of parallelogram – which one is it? *(See p.241-243 for definitions of quadrilaterals.)*



Hm, how can we prove that $\overline{BU} \cong \overline{UC}$? Well, if $\triangle BUC$ were an isosceles triangle with base $\overline{BC}$, then we would know $\overline{BU} \cong \overline{UC}$, right? And one way to prove that a triangle is isosceles is to prove that the base angles are congruent. In this case, that would be $\angle UBC \cong \angle UCB$ (make sure you see them on the diagram). So that's our new goal!

Hm, how can we prove that $\angle UBC \cong \angle UCB$? Well, we know that BUCK is a parallelogram, which means that opposite angle are congruent – in other words: $\angle UBK \cong \angle UCK$. We are also told that $\overline{BC}$ bisects these two, big, congruent angles. "Bisect" means to cut in half, of course! Remember our pizzas and Division Property from p. 80? If we cut two congruent angles into halves, then all the little halves must also be congruent! Since $\angle UBC$ is half of $\angle UBK$, and $\angle UCB$ is half of $\angle UCK$, we now know that, for example, $\angle UBC \cong \angle UCB$. Great!

Now, why did we want to prove $\angle UBC \cong \angle UCB$? Oh yeah – because those are two angles in the triangle $\triangle BUC$, which means $\triangle BUC$ is an isosceles triangle with base $\overline{BC}$, and that means its congruent legs are $\overline{BU} \cong \overline{UC}$, which is what we were trying to prove all along! Notice that we never even have to use the word "isosceles" because of our handy "if angles, then sides" rule (see p. 148 to review this).

Pant, pant. Done with the first part! Here's this proof in two-column form:

♥Proof♥

Statements	**Reasons**
1. $BUCK$ is a parallelogram	1. Given
2. $\angle UBK \cong \angle UCK$	2. If a $\parallel$-ogram, then opp $\angle$'s are $\cong$.
3. $\overline{BC}$ bisects $\angle UBK$ & $\angle UCK$	3. Given
4. $\angle UBC \cong \angle UCB$	4. Division property (Halves of congruent angles are congruent)
5. $\therefore \overline{BU} \cong \overline{UC}$	5. If angles, then sides. (In step 4, we discovered that $\triangle BUC$ is an isosceles $\triangle$)

Now, to answer the second part of the question: What else do we know about the sides of BUCK? **Well, we know that opposite sides are $\cong$ on all parallelograms**, which means $\overline{BU} \cong \overline{CK}$ and $\overline{UC} \cong \overline{BK}$.

Above, we've just proven that $\overline{BU} \cong \overline{UC}$. And now we could use the Transitive Property to prove that since $\overline{BU} \cong \overline{CK}$ and $\overline{BU} \cong \overline{UC}$, we can conclude $\overline{CK} \cong \overline{UC}$. In fact, the Transitive Property tells us now that all the sides are congruent! And that's the definition of a rhombus (see p. 242). Yep, we've proven that ***BUCK*** **is a rhombus**!