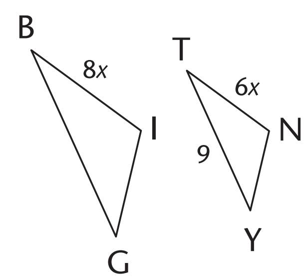
Girls Get Curves

Solution Guide for Chapter 17

Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

DTM from p. 291



2. Given $\triangle BIG \sim \triangle TNY$, $BI = 8x$, $TN = 6x$, $TY = 9$. Find BG.

These are similar triangles, which means that the ratio of their sides are equal. In other words: $\frac{BI}{TN} = \frac{BG}{TY}$. Filling in the values we know, this becomes: $\frac{8x}{6x} = \frac{BG}{9}$. First, let's notice that we can cancel the x from the top and bottom of the left-hand fraction (check out chapter 4 in "Hot X: Algebra Exposed" if you're not sure why we are allowed to do that!), and now our proportion becomes: $\frac{8}{6} = \frac{BG}{9}$. Nice! How can we solve for BG? We can cross-multiply to get an equivalent equation! Cross multiplying, we get $72 = 6BG$, and dividing both sides by 6 we get: $\boldsymbol{BG = 12}$. Does it make sense that BG would be 12? Yep!

Answer: $\boldsymbol{BG = 12}$

3. Your best friend is exactly 5 feet, 3 inches tall, and in a full-length picture, she appears to be 7 inches tall. If her head appears to be 1 inch tall, how tall is her head in real life? (Hint: Watch the units!)

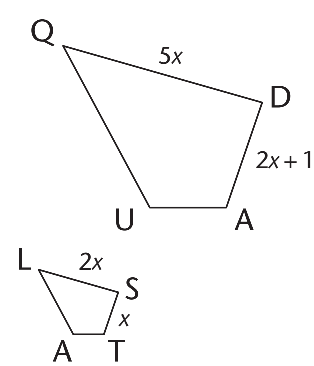
Okay, so we've been given inches AND feet. We'd better decide on one unit and be consistent while we solve this! Should we use inches or feet? Let's use inches, so we don't end up with decimals or fractions for the actual measurements.
Since 1 foot = 12 inches, that means 5 feet = $12 \cdot 5 = 60$ inches. So "5 feet 3 inches" would be **63** inches. With me so far?

Now that we have consistent units, let's set up the proportion between "real life" and "appears in picture." It'll look like this: $\frac{\textit{head in real life}}{\textit{body in real life}} = \frac{\textit{head in picture}}{\textit{body in picture}}$. Filling in what we know, this becomes: $\frac{\textit{head in real life}}{63} = \frac{1}{7}$. Hm, let's use the variable "*h*" to stand for "head in real life," and this becomes: $\frac{h}{63} = \frac{1}{7}$. Now we solve for *h*!

Cross-multiplying, we get: $7h = 63 \cdot 1$ → $7h = 63$ → $\boldsymbol{h = 9}$. So her head is 9 inches tall in real life. Sounds like a reasonable answer!

Answer: 9 inches

4. In the QUAD-LATS diagram, if we are also told that $QU = 8$, find LA.



In problem #1, we learned that $QD = 10$, and we're allowed to use this here. Let's set up a proportion between these two similar shapes! $\overline{QU}$ corresponds to $\overline{LA}$, so that's good news. (There's more than one way to do this, by the way!) Since they are similar shapes, we know that: $\frac{QU}{LA} = \frac{QD}{LS}$, right?

Now let's fill in what we know, and the proportion becomes: $\frac{8}{LA} = \frac{5x}{2x}$. On the right side of this equation, we can cancel the "*x*" factor from top and bottom and we get: $\frac{8}{LA} = \frac{5}{2}$.

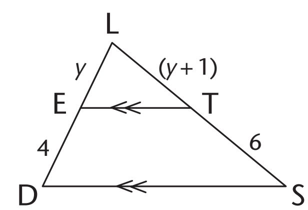
Finally, we'll cross-multiply to get this equivalent equation: 16 = 5*LA*.

Dividing by 5, we get: $LA = \frac{16}{5}$.

This *is* the value we were looking for, but when an answer is an improper fraction, it's usually best to convert it into a mixed number: $\frac{16}{5} = \mathbf{3\frac{1}{5}}$. (Check out the MAD face method on p. 45 of "Math Doesn't Suck" to review converting to mixed numbers.) There's no way to further simplify this, so we're done!

Answer: $\boldsymbol{LA = 3\frac{1}{5}}$

5. Given $\overline{ET} \parallel \overline{DS}$. If *DE* = 4, *TS* = 6, *LE* = *y*, and *LT* = *y* + 1, find *LS*.



We can apply the same logic we did in the example on p. 290:

Here, since $\overline{ET} \parallel \overline{DS}$, then because of "If || lines, then corr. $\angle$'s are $\cong$," we know that $\angle LET \cong \angle LTE$ and $\angle LDS \cong \angle LSD$. So by AA~ (see p.289), we know that we have two similar triangles: $\triangle DLS \sim \triangle ELT$. Great progress! Since we have similar triangles, we know that we can make a proportion out of their corresponding sides: $\frac{LE}{LD} = \frac{LT}{LS}$. In this case, we want to be careful and pay extra attention before we fill in the known values – notice that the sides of the bigger triangle require us to *add together* the lengths we're given. Filling in what we know, this becomes: $\frac{y}{y+4} = \frac{(y+1)}{(y+1)+6}$.

Whoa, scary! But let's not be afraid - we can totally do this! First off, we simplify the bottom of the right hand fraction by dropping the parentheses and adding the 1 and the 6 together and we get: $\frac{y}{y+4} = \frac{(y+1)}{y+7}$. You might ask, what's the difference between having parentheses or not? In this case, there's no difference! We could totally write it like this: $\frac{y}{y+4}=\frac{y+1}{y+7}$ or even $\frac{y}{(y+4)}=\frac{(y+1)}{(y+7)}$; they mean the same thing. Simply by BEING the numerator or denominator of a fraction, it's as if you have parentheses around yourself. However, as soon as we cross-multiply, we need to actually put the parentheses on! Here's how that looks:

$$\frac{y}{y+4}=\frac{y+1}{y+7}$$

cross multiplying →

$$y(y + 7) = (y + 4)(y + 1)$$

And now we use a little distribution to simplify this further:

$$y^2 + 7y = y^2 + y + 4y + 4$$

Still looks pretty scary – but wait! We can subtract y^2 from both sides! Also combining like terms on the right side, this whole thing becomes:

$$7y = 5y + 4$$

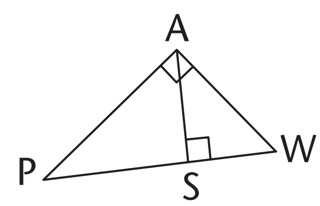
Ah, MUCH better! Subtracting $5y$ from both sides, we get: $2y = 4$, so $y = 2$.
Careful! That's not the answer yet. The question asked for *LS*. Since $LS = (y + 1) + 6$, that means:

$$\begin{aligned} LS &= (y + 1) + 6 \\ &= (2 + 1) + 6 \\ &= (3) + 6 \\ &= \mathbf{9} \end{aligned}$$

Pant, pant. Done! And looking at the diagram, does this answer seem reasonable? Yep!

Answer: *LS* = 9

DTM from p. 296-297



2. Given: $\triangle PAW$ is a right triangle, with right angle $\angle PAW$ and altitude $\overline{AS}$. Prove $\triangle PSA \sim \triangle PAW$

Okay, so we're supposed to prove that the big triangle is similar to the medium-sized triangle, right? Since we're told that $\overline{AS}$ is an altitude, we know that the medium triangle is indeed a right triangle, with $\angle PSA$ being the right angle, right? So that means "A" in the big triangle will correspond with the "S" in the medium-sized triangle. Since all right angles are congruent, we know that $\angle PAW \cong \angle PSA$.

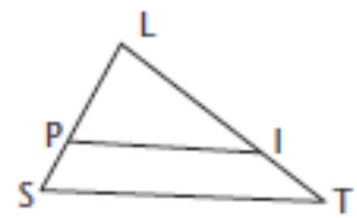
These two triangles also share $\angle P$ in common – and by the Reflexive Property, obviously, $\angle P \cong \angle P$, right? AA$\sim$ says all we need is for *two pairs* of angles to be congruent in order to prove that two triangles are similar. And that's what we've done!

♥Proof♥

Statements	**Reasons**
1. $\triangle PAW$ is a right triangle, with right angle $\angle PAW$.	1. Given
2. $\overline{AS}$ is an altitude	2. Given
3. $\angle PSA$ is a right angle	3. Definition of altitude (altitudes create right angles)
4. $\angle PAW \cong \angle PSA$	4. All right angles are congruent (Gimmie an "A"!)
5. $\angle P \cong \angle P$	5. Reflexive Property (Gimmie another "A"!)
6. $\therefore \triangle PAW \sim \triangle PSA$	6. AA$\sim$

3. See the SPLIT diagram to the right. Given: $\overline{PI} \parallel \overline{ST}$.

Prove: $\frac{PL}{SL} = \frac{IL}{TL}$ in a two-column proof.



Looking at the small triangle and the big triangle, we're being asked to prove that corresponding sides are proportional. Well, if we can prove that the two triangles are similar, then we'll have proved just that!

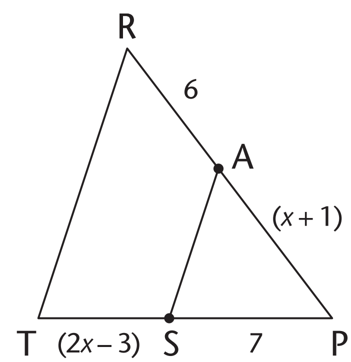
If we can show that two sets of corresponding angles are congruent, we'll be set. By the Reflexive property, $\angle L \cong \angle L$, and to get another set, we'll use very similar logic as the problem on p. 290 – since $\overline{PI} \parallel \overline{ST}$, we know that corresponding angles are congruent! Looking just at the left-hand escalator, $\angle LPI$ & $\angle LST$ are corresponding angles, and because $\overline{PI} \parallel \overline{ST}$, we know they must be congruent. We could use the other escalator to get the third pair of angles congruent, but we don't need it!

♥Proof♥

Statements	**Reasons**
1. $\overline{PI} \parallel \overline{ST}$	1. Given
2. $\angle LPI \cong \angle LST$	2. If $\parallel$, then corr. $\angle$'s are $\cong$. (Gimmie an "A"!)
3. $\angle L \cong \angle L$	3. Reflexive Property (Gimmie another "A"!)
4. $\triangle LPI \sim \triangle LST$	4. AA $\sim$
5. $\therefore \ \frac{PL}{SL} = \frac{IL}{TL}$	5. If two $\triangle$'s are $\sim$, then corresponding sides are proportional.

The above proof was actually the beginning of the Side-Splitter theorem. See the "A proof of the Side-Splitter Theorem" PDF on the Extras page of GirlsGetCurves.com for the rest of that proof!

4. **The Midpoint Theorem** says that if a segment connects two sides of a triangle at their midpoints, then that segment is parallel to the third side and is half the length of the third side. Given: $RA = 6$, $AP = (x + 1)$, $TS = (2x - 3)$, $SP = 7$, and S is the midpoint of $\overline{TP}$. Use a paragraph proof to show that $\overline{TR} \parallel \overline{SA}$ and that $\overline{SA}$ is half the length of $\overline{TR}$. *(Hint: If S is the midpoint of* $\overline{TP}$*, then what two segments must be congruent? Use that to find x, and then find AP. What does this tell us about A? Now use the Midpoint Theorem!)*



Hm, looking closely at the Midpoint Theorem above, it seems that if the segment $\overline{SA}$ connects two sides of the triangle at their midpoints (so, if A is the midpoint of $\overline{RP}$ and if S is the midpoint of $\overline{TP}$), then that segment, $\overline{SA}$, would automatically be parallel to the third side of the triangle, which is $\overline{TR}$, and $\overline{SA}$ would be half the length of $\overline{TR}$. Do you see how we just applied the above theorem to this diagram? Make sure you do before reading on!

Now – if we understand that, then all we have to do on this diagram is prove that A is the midpoint of $\overline{RP}$ and S is the midpoint of $\overline{TP}$! Because then we'd have proven that $\overline{SA}$

connects two sides of the triangle at their midpoints, just in the theorem. So that's our new goal! Let's do it:

Paragraph proof:

Since we are given that *S* is the midpoint of $\overline{TP}$, that means *TS* = *SP*, which means 2*x* – 3 = 7. With me so far? Solving this, we get *x* = 5. And since *AP* = *x* + 1, that means *AP* = 6.

Since *AP* = 6 and *RA* = 6, we know that *A* must be the midpoint of $\overline{RP}$, right? So we have a segment, $\overline{AS}$, connecting the midpoints of two sides of $\triangle TRP$. Now we can apply the Midpoint Theorem, which tells us that if $\overline{AS}$ connects the midpoints of $\overline{RP}$ and $\overline{TP}$, then $\overline{TR} \parallel \overline{SA}$ and that $\overline{SA}$ is half the length of $\overline{TR}$. (Pant, pant.) Done!

$\therefore$ $\overline{AS}$ **connects the midpoints of** $\overline{RP}$ **and** $\overline{TP}$**, then** $\overline{TR} \parallel \overline{SA}$ **and** $\overline{SA}$ **is half the length of** $\overline{TR}$