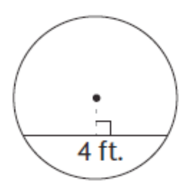
Girls Get Curves

Solution Guide for Chapter 18

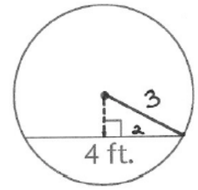
Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

DTM from p. 310-311

2. The diameter of a circle is 6 ft across. A chord measures 4 ft. How far is the chord from the center? *(Hint: Where would it be most helpful to draw the radius?)*



Let's take the hint and draw a radius in, and let's draw it from the center point to one of the endpoints of the chord – that way we can make a right triangle, which should help us get more information. Remember – drawing in radii is almost always a great idea!



Now, once we have our right triangle, we can start filling in more lengths – after all, we know if the diameter of the circle is 6, then the radius measures 3, right? Also, the Symmetrical Bulldog Puppy Mouth theorem from p. 306 tells us that the perpendicular segment drawn to the chord *must* also bisect that chord, and since the chord is 4 ft long, so we can write “2” as half its length - for the bottom side of our right triangle. Nice progress! Now we can use the Pythagorean theorem to find the third side of the right triangle.

$$a^2 + b^2 = c^2$$

Filling in $a = 2$ and $c = 3$, this becomes:

$$2^2 + b^2 = 3^2$$

$$\rightarrow 4 + b^2 = 9$$

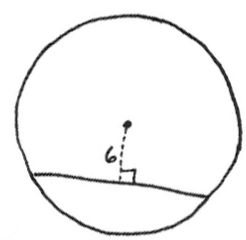
$$\rightarrow b^2 = 5$$

$$\rightarrow b = \sqrt{\mathbf{5}}$$

And that’s the distance from the center to the chord, so we’re done!

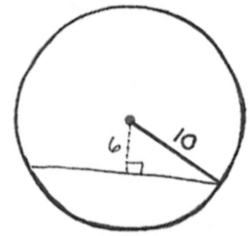
Answer: $\sqrt{\mathbf{5}}$ ft.

3. A circle has a chord whose distance is 6 inches from the center, and the radius is 10 inches. What is the length of the chord? *(Hint: Draw the circle, chord, and distance first. Then fill in the radius to make a right triangle.)*



Ok. Let’s start by drawing our very own circle, as the hint suggests. It doesn’t have to be perfect; just good enough so we can tell what’s going on!

Now it seems more clear where we should draw the radius so that we can get a right triangle. Let's do it! And of course we can fill in the radius length we were given: 10. How about that? We're ready to find that missing length on the triangle:



$$a^2 + b^2 = c^2$$

Filling in $a = 6$ and $c = 10$, this becomes:

$$6^2 + b^2 = 10^2$$

$$\rightarrow 36 + b^2 = 100$$

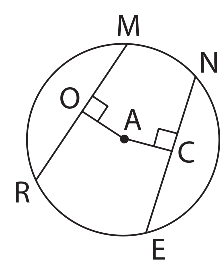
$$\rightarrow b^2 = 64$$

$$\rightarrow b = 8$$

But that's not our final answer! This is the length of the missing side on our triangle. The problem asked for the length of the entire chord. Because of the Symmetrical Bulldog Puppy Mouth theorem, we know that the perpendicular line from the center of the circle to the chord must also bisect the chord, and that means the entire chord is exactly twice the length we've found. In other words, the chord is 16 inches long!

Answer: 16 in.

4. In ROMANCE, $\overline{RM} \cong \overline{EN}$. $OA = 2x + 5$, $AC = 4x + 1$, and $RM = 8x - 3$. Find EC. *(Hint: Use a Congruent-Chord (Twin Puppy) Theorem and the first two equations to find x, then evaluate RM, and then find EC.)*



On p. 308, the Converse of the Congruent-Chord (Twin Puppy) Theorem tells us that if two chords are congruent, then those two chords are equidistant to the center. On this diagram, since $\overline{RM} \cong \overline{EN}$, this theorem tells us that each chords' distance to the center will be the same, in other words, this means that $OA = AC$. Since we've been given expressions for both OA and AC, we can set them equal to each other (since we now know they are equal!) and solve for x:

$$OA = AC$$

$$\rightarrow 2x + 5 = 4x + 1$$

$$\rightarrow 5 = 2x + 1$$

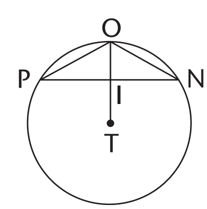
$$\rightarrow 4 = 2x$$

$$\rightarrow x = 2$$

Great progress! Following the hint further, we can now find RM. Since we've been told $RM = 8x - 3$, we can fill in $x = 2$ and we get: $RM = 8(2) - 3 = 16 - 3 =$ **13**.

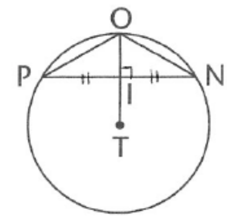
Since we know that $RM = EN$, that means $EN = 13$, too. And because of the Symmetrical Bulldog Puppy Mouth theorem, we know that since AC is perpendicular to EN, it also must bisect it, and that means EC is half the length of EN. In other words, $EC = 6.5$. Pant, pant. Done!

Answer: *EC* = 6.5



5. For the POINT diagram, given $\odot P$ and $\overline{TO} \perp \overline{PN}$, prove $\angle P \cong \angle N$ in a two-column proof.

We're given a chord with a perpendicular radius, so even though the tooth (right angle marker) wasn't drawn in for us, we could immediately think of the Symmetrical Bulldog Puppy Mouth theorem! It tells us that that radius is *bisecting* the chord, which means that $\overline{PI} \cong \overline{IN}$. How can we use this to prove that $\angle P \cong \angle N$? Hm, if those two little triangles were congruent (reflections of each other, across that radius!), then CPCTC would tell us

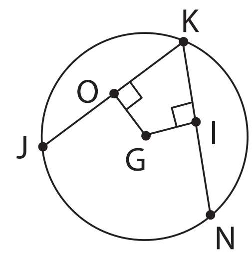


$\angle P \cong \angle N$, right? So, can we get $\triangle POI \cong \triangle NOI$? Yep! The Reflexive property tells that $\overline{OI} \cong \overline{OI}$ (Gimmie an "S"!), we've been told that $\overline{TO} \perp \overline{PN}$, which means those little triangles are both right triangles, so: $\angle OIP \cong \angle OIN$ (Gimmie an "A"!), and above, the Symmetrical Bulldog Puppy Mouth Theorem told us that $\overline{PI} \cong \overline{IN}$ (Gimmie an "S"!) Looking at the diagram, this gives us SAS, so yes! We've proven that $\triangle POI \cong \triangle NOI$. And now CPCTC tells us that $\angle P \cong \angle N$. Yay! Here it is, in two-column format:

♥Proof♥

Statements	**Reasons**
1. $\odot P$, $\overline{TO} \perp \overline{PN}$	1. Given
2. $\overline{TO}$ bisects $\overline{PN}$	2. If a radius is $\perp$ to a chord, then it bisects the chord. (Symmetrical Bulldog Puppy Mouth Theorem!)
3. $\overline{PI} \cong \overline{IN}$	3. Def. of bisect: If a seg is bisected, it's divided into two $\cong$ seg's. (Gimmie an "S"!)
4. $\angle OIP$ & $\angle OIN$ are right angles	4. When two segments are $\perp$, they create right angles.
5. $\angle OIP \cong \angle OIN$	5. All right angles are $\cong$. (Gimmie an "A"!)
6. $\overline{OI} \cong \overline{OI}$	6. Reflexive Property (Gimmie an "S"!)
7. $\triangle POI \cong \triangle NOI$	7. SAS (3, 5, 6)
8. $\therefore \angle P \cong \angle N$	8. CPCTC

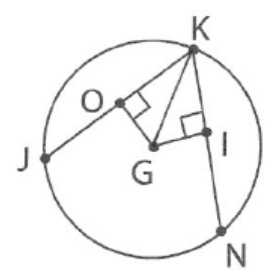
6. For the JOKING diagram, given $\odot G$, $\overline{GO} \perp \overline{KJ}$, $\overline{GI} \perp \overline{KN}$, $\overrightarrow{KG}$ bisects $\angle JKN$. Prove: $\overline{KJ} \cong \overline{KN}$.



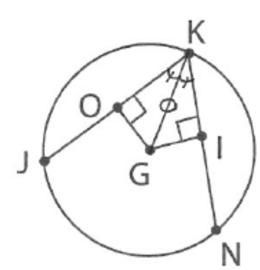
Hmmm. How do we start? Let's work backwards on this one.

We're being asked to prove that two chords are congruent, right?

Well, what Rule do we know that tells us something about congruent chords? The Twin Puppy Dog (Congruent Chord) Theorem, of course! It states that if two chords are equidistant to the center, then the chords are congruent. What are the distances from the chords to the center? *OG & IG*! *(Since we see those right angle markers, we know that OG is the* <u>*distance*</u> *from the point G to the chord* $\overline{JK}$*, and we know that IG is the* <u>*distance*</u> *from the point G to the chord* $\overline{KN}$*.)* So, can we prove that the two chords are equidistant to the center? In other words, can we prove that $\overline{OG} \cong \overline{IG}$? Hm, if only we had some congruent triangles that had $\overline{OG}$ & $\overline{IG}$ as corresponding sides.



Oh, but we CAN have some! Let's draw in the segment $\overline{KG}$ and lookie there – those two little triangles sure look like they could be congruent. How can we prove that $\triangle KOG \cong \triangle KIG$? Well, they are both right triangles (Gimmie an "A"!), and since we were told that $\overrightarrow{KG}$ bisects $\angle JKN$, that will give us two congruent angles: $\angle GKO \cong \angle GKI$ (Gimmie an "A"!) and the good 'ol Reflexive property gives us the shared side, $\overline{KG}$. (Gimmie an "S"!) And that gives us AAS. Ok, phew!

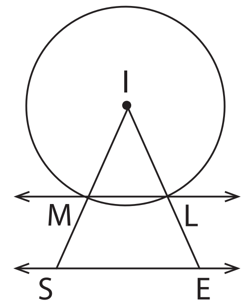


So, unraveling this "backwards" logic, we draw in the segment $\overline{KG}$, then prove that those two little triangles are congruent, then use CPCTC to prove $\overline{OG} \cong \overline{IG}$, which are the distances from the center to $\overline{KJ}$ & $\overline{KN}$. And then the Twin Puppy Dog Theorem tells us that since these two distances are the same, that means the chords must also be congruent. Pant, pant! Nice job following that! Here it is in two-column format:

♥Proof♥

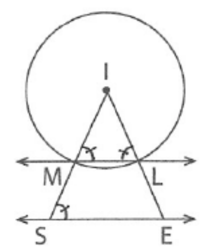
Statements	**Reasons**
1. $\odot G$, $\overline{GO} \perp \overline{JK}$, $\overline{GI} \perp \overline{KN}$	1. Given (The $\perp$'s tell us that $\overline{GO}$ marks the distance from G to $\overline{KJ}$ and that $\overline{GI}$ marks the distance to $\overline{KN}$.)
2. Draw $\overline{KG}$	2. Two points determine a line.
3. $\angle KOG$ & $\angle KIG$ are right $\angle$'s	3. $\perp$ segments create right $\angle$'s.
4. $\angle KOG \cong \angle KIG$	4. All right angles are $\cong$. (Gimmie an A!)
5. $\overline{KG}$ bisects $\angle JKN$	5. Given
6. $\angle GKO \cong \angle GKI$	6. If a ray bisects an $\angle$, it divides it into two $\cong$ $\angle$'s. (Gimmie an A!)
7. $\overline{KG} \cong \overline{KG}$	7. Reflexive property (Gimmie an S!)
8. $\triangle KOG \cong \triangle KIG$	8. AAS (4, 6, 7) Yay, congruent triangles!
9. $\overline{OG} \cong \overline{IG}$	9. CPCTC (Great, we proved the distances from the center to the chords are the same!)
10. $\therefore \overline{KJ} \cong \overline{KN}$	10. If two chords are equidistant from the center, then the chords are congruent. (Twin Puppy Dog Theorem)

7. For the SMILE diagram, Given $\odot I$ and $\overline{SE} \parallel \overline{ML}$, prove $\angle MSE \cong \angle LES$.

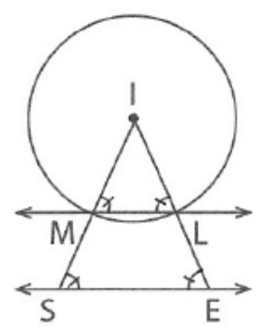


Hm, we're supposed to prove that those lowest angles are congruent. Well, since we've been given that $\overline{SE} \parallel \overline{ML}$, this diagram sure has that escalators-at-the-mall feel to it, doesn't it? We know radii on a circle are always congruent, so that means $\overline{IM} \cong \overline{IL}$. And that means the small triangle, $\triangle MIL$, must be isosceles, which in turn means that its base angles must be congruent: $\angle IML \cong \angle ILM$. Those aren't the angles we want to prove are congruent, but now it's time to use what we know about malls…

First let's look at the left side of the diagram: Since $\overline{SE} \parallel \overline{ML}$ and $\overline{SM}$ is a transversal (escalator), that means the two corresponding angles $\angle IML$ & $\angle MSE$ must be congruent, right? (If lines are parallel, then corr. $\angle$'s are $\cong$.) Let's draw this in so far. See how all three of these marked angles are forced to be congruent?



Similar logic on the right side with the $\overline{LE}$ escalator tells us that $\angle LES \cong \angle ILM$. Remember the four-element version of the Transitive Property from p. 53 of Girls Get Curves? It says that if two $\cong$ $\angle$'s ($\angle IML \cong \angle ILM$) are each $\cong$ to two *other* $\angle$'s ($\angle MSE$ and $\angle LES$) then those two *other* $\angle$'s are $\cong$. In other words, it must be true that $\angle MSE \cong \angle LES$. Done!



Think of it this way: If Becky & Cathy are the same height (the little triangle's base angles), Becky & Jessica are the same height (the left side escalator angles) and then we're also told that Cathy & Madison are the same height (the right side escalator angles), that means that they are

ALL the same height, right? Which means, certainly, that Madison and Jessica are the same height (the big triangle's base angles).

Let's write this out!

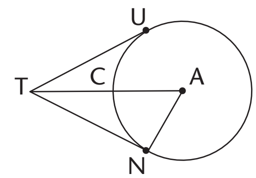
♥Proof♥

Statements	**Reasons**
1. $\odot I$	1. Given
2. $\overline{IM} \cong \overline{IL}$	2. All radii of a circle are $\cong$.
3. $\triangle MIL$ is isosceles	3. If two sides of a triangle are $\cong$, then the triangle is isosceles.
4. $\angle IML \cong \angle ILM$	4. If a triangle is isosceles, then its base $\angle$'s are $\cong$.
5. $\overline{SE} \parallel \overline{ML}$	5. Given
6. $\angle MSE \cong \angle IML$	6. If lines are parallel, then corr. $\angle$'s are $\cong$. (Remember the mall)
7. $\angle LES \cong \angle ILM$	7. Same as above (but using a different escalator!)
8. $\therefore \angle MSE \cong \angle LES$	8. Transitive Property – four element version [If two $\cong$ $\angle$'s ($\angle IML \cong \angle ILM$) are each $\cong$ to two *other* $\angle$'s ($\angle MSE$ and $\angle LES$) then those two *other* $\angle$'s are $\cong$.]

(Notice we could have combined steps 3 & 4 with the "if sides, then angles" shortcut!)

DTM from p. 315-316

2. In TUCAN, $\overline{TU}$ and $\overline{TN}$ are tangent to $\odot A$. The radius of $\odot A$ is 6 cm. $TU = 2x + 2$, and $TN = 3x - 1$. Find *TN*, *TA*, and *TC*. *(Hint: After finding x, use the Pythagorean Theorem!)*



Sure looks like a bird beak to me! So, according to the Two-Tangent (Bird Beak!) Theorem, those tangent segments that intersect at the point *T* must be equal in length. In other words: $TU = TN$. And since we have expressions for both lengths, we can fill them in and solve:

$$TU = TN$$

$$\rightarrow\ 2x + 2 = 3x - 1$$

$$\rightarrow\ 2x + 3 = 3x$$

$$\rightarrow\ \mathbf{3} = x$$

Okay, now that we know that $x = 3$, what good does it do us? Well, we can find the lengths of *TU* and *TN*, by plugging 3 in for *x*! That means

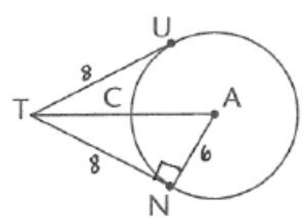
$$TU = 2x + 2 = 2(3) + 2 = 6 + 2 = \mathbf{8}$$

And *TN* should also equal 8, but let's double check:

$$TN = 3x - 1 = 3(3) - 1 = 9 - 1 = \mathbf{8}$$

Great! So we can fill those lengths into the diagram.

And of course we can also fill in the length of $\overline{NA}$, a radius of the circle (which we were told is **6 cm**). Finally, because of the Bicycle Rule from p. 313, we can add in a right angle



marker where the tangent $\overline{TN}$ intersects the radius $\overline{NA}$! And… now we have a right triangle with two of the legs filled in, which means we can find the hypotenuse, no problem – we'll just apply the good 'ol Pythagorean theorem!

$$a^2 + b^2 = c^2$$

$$\rightarrow TN^2 + NA^2 = TA^2$$

Filling in $TN = 8$ and $NA = 6$, this becomes:

$$8^2 + 6^2 = TA^2$$

$$\rightarrow 64 + 36 = TA^2$$

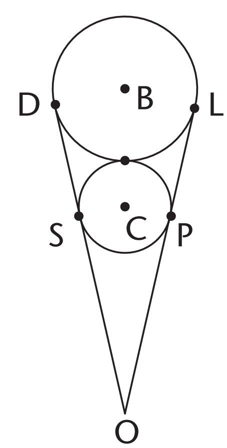
$$\rightarrow 100 = TA^2$$

$$\rightarrow \boldsymbol{TA = 10}$$

Ok, we've found a bunch of stuff, but let's make sure we remember what the question was asking for: It wanted us to find *TN*, *TA*, and *TC*.

Well, we found *TN* earlier: ***TN* = 8**, and we just found ***TA* = 10**. What about *TC*? Well, from the diagram we can assume that $TC = TA - CA$, right? And *CA* is just a radius, so it measures 6 cm. That means: $TC = 8 - 6 = \mathbf{2}$. Phew!

Answer: *TN* = 8 cm, *TA* = 10 cm, *TC* = 2 cm



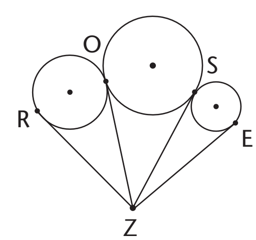
3. In the double scoop (DBLSCOP) figure to the right, given that $\overline{DO}$ & $\overline{LO}$ are common tangents to $\odot B$ & $\odot C$ at points *D*, *S*, *L* & *P*, prove $\overline{DS} \cong \overline{LP}$.

Ah, we have two bird beaks, so we can apply the Two-Tangent (Bird Beak) Theorem twice! We have one big long bird beak and one smaller one. Now, how is that going to lead us to proving $\overline{DS} \cong \overline{LP}$? Well, a little subtraction and we'll be golden – you see, those two segments are just the leftovers we get when we subtract the shorter bird beak sides from the long ones. See it? The long bird beak tells us that $\overline{DO} \cong \overline{LO}$, and the smaller bird beak tells us $\overline{SO} \cong \overline{PO}$. And if we subtract two congruent segments from two other congruent segments, we still end up with two congruent segments – that's the good 'ol Subtraction Property from Chapter 5! Let's write it out:

♥Proof♥

Statements	**Reasons**
1. $\overline{DO}$ & $\overline{LO}$ are common tangents to $\odot B$ & $\odot C$ at points D, S, L & P	1. Given
2. $\overline{DO} \cong \overline{LO}$	2. Two-Tangent theorem
3. $\overline{SO} \cong \overline{PO}$	3. Two-Tangent theorem
4. $\therefore \overline{DS} \cong \overline{LP}$	4. Subtraction Property (If two $\cong$ seg's are subtracted from two $\cong$ seg's, the differences are $\cong$.)

4. In the ROSEZ diagram, all segments are tangent to the circles they touch. Is it necessarily true that $\overline{RZ} \cong \overline{EZ}$? Explain one way or the other, in a paragraph proof.

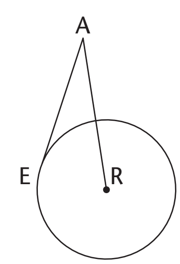


Proof: Yes! In face, we can show $\overline{RZ} \cong \overline{EZ}$ by just using the Bird Beak Two-Tangent theorem and the Transitive Property a few times! Looking at just the leftmost "bird beak," by the Two-Tangent Theorem, It must be true that $\overline{RZ} \cong \overline{OZ}$. Looking at just the center "bird beak", for the same reason, we know $\overline{OZ} \cong \overline{SZ}$. The Transitive property now tells us that $\overline{RZ} \cong \overline{SZ}$. Looking at just the rightmost "bird beak," we know that $\overline{SZ} \cong \overline{EZ}$. Applying the Transitive property to these last two statements, we get: $\overline{RZ} \cong \overline{EZ}$. It's kind of interesting that it doesn't matter how big the circles are; in this type of configuration, the tangent segments are forced to all be congruent to each other!

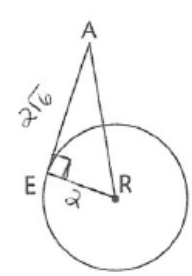
$\therefore \overline{RZ} \cong \overline{EZ}$

DTM from p. 320-321

2. In the EAR diagram, $\overline{EA}$ is tangent to $\odot R$, $EA = 2\sqrt{6}$, and the radius of $\odot R$ is 2. Find RA.



What should we ALWAYS do when we see a circle diagram? Fill in what we know, and look for good places to draw in radii! And the segment $\overline{EA}$ is tangent at the point E, so let's draw in $\overline{ER}$, which a radius and also touches a tangent line – and that means we've created a right angle! So let's draw a right angle marker, too. Since the radius of $\odot R$ is 2, now we know that $ER = 2$, and we can also fill in that $EA = 2\sqrt{6}$. And lookie there, now we have a right triangle with two lengths filled in and Pythagoras can finish it off for us!



$$a^2 + b^2 = c^2$$

$$\rightarrow ER^2 + EA^2 = RA^2$$

Filling in $ER = 2$ and $EA = 2\sqrt{6}$, this becomes:

$$2^2 + (2\sqrt{6})^2 = RA^2$$

$$\rightarrow 4 + \left(2^2 \cdot \sqrt{6}^2\right) = RA^2$$

$$\rightarrow 4 + (4 \cdot 6) = RA^2$$

$$\rightarrow 4 + 24 = RA^2$$

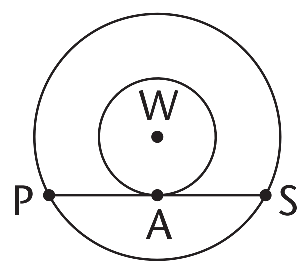
$$\rightarrow 28 = RA^2$$

$$\rightarrow RA = \sqrt{28}$$

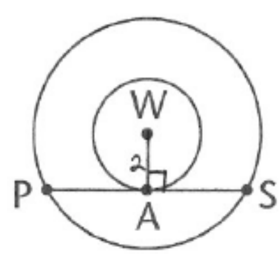
$$\rightarrow \boldsymbol{RA = 2\sqrt{7}}$$

And that's what we were asked to find!

Answer: $\boldsymbol{RA = 2\sqrt{7}}$

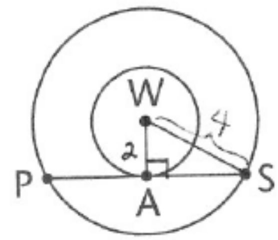


3. In PAWS, concentric circles both have center W, with radii of 2 and 4. Find PS.



And…the first thing we always do? Draw in helpful radii! In this case, there are two circles we can draw in radii for, and we want to think carefully about which ones would be most helpful. Hm. For the smaller circle, the only point marked on the edge is A, so let's draw in the radius $\overline{WA}$. And we know $WA = 2$, so we can fill that in. And hey, since PS is tangent to the smaller circle at A, that means $\overline{PS}$ and $\overline{WA}$ create a right angle, so let's mark that, too. Great progress!

For the larger circle, it looks like it could make sense to draw radii to the points P and/or S. But as it turns out, we only need one of them! Let's draw $\overline{WS}$, and we know its length is 4 so we can fill that in, too. Yep, we get a right triangle with a leg and the hypotenuse!



Let's use Pythagoras to find the missing leg, AS:

$$a^2 + b^2 = c^2$$

$$WA^2 + AS^2 = WS^2$$

Filling in $WA = 2$ and $WS = 4$, this becomes:

$$2^2 + AS^2 = 4^2$$

$$\rightarrow 4 + AS^2 = 16$$

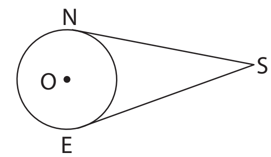
$$\rightarrow AS^2 = 12$$

$$\rightarrow AS = 2\sqrt{3}$$

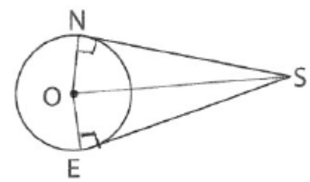
Now, we could either go through the identical process on the left side of the diagram and discover that indeed, PA also equals $2\sqrt{3}$, and then add them together to get $PS = 2\sqrt{3} + 2\sqrt{3} = \mathbf{4\sqrt{3}}$. Or, we could notice that the Symmetrical Puppy Dog Theorem from p. 306 tells us that because $\overline{WS}$ creates a right angle with $\overline{PS}$, $\overline{WA}$ must also *bisect* $\overline{PS}$, meaning that $PA = PS$. Either way, we know that $\overline{PS}$ has twice the length of $\overline{AS}$, which means $PS = 2(2\sqrt{3}) = \mathbf{4\sqrt{3}}$

Answer: *PS* = $\mathbf{4\sqrt{3}}$

4. Proof of bird beak theorem: In NOSE, given $\odot O$ and tangent segments $\overline{NS}$ & $\overline{ES}$, prove that $\overline{NS} \cong \overline{ES}$.



The explanation is fully worked out on p. 318-319 (which includes drawing in those two radii touching the tangent lines, and the segment $\overline{OS}$!) Here's how the two-column proof looks!



♥Proof♥

Statements	**Reasons**
1. $\overline{NS}$ & $\overline{ES}$ are tangent to $\odot O$	1. Given
2. Draw $\overline{ON}, \overline{OE}, \overline{OS}$	2. Two points determine a line.
3. $\overline{ON} \perp \overline{NS}$ and $\overline{OE} \perp \overline{ES}$	3. Tangent lines are $\perp$ to the radii that touch the tangent points. (Bicycle wheel rule)
4. $\angle OES$ & $\angle ONS$ are right angles	4. If segments are $\perp$, then they form right angles.
5. $\overline{ON} \cong \overline{OE}$	5. Radii are $\cong$. (the L in HL!)
6. $\overline{OS} \cong \overline{OS}$	6. Reflexive Property (the H in HL!)
7. $\triangle NOS \cong \triangle EOS$	7. HL (5, 6)
8. $\therefore \overline{NS} \cong \overline{ES}$	8. CPCTC