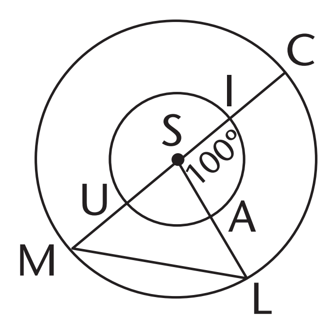
Girls Get
Curves

Solution Guide for Chapter 19

Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

DTM from p. 329-330



2. The measure of $\widehat{AI}$ is the same as the measure of its central angle, which is 100°. And the same is true for $\widehat{CL}$! Even though $\widehat{CL}$ looks bigger, its *measure* is defined by the *central angle* – it's kind of like measuring what fraction of the entire circumference the arc is!

Answer: Both measure 100°.

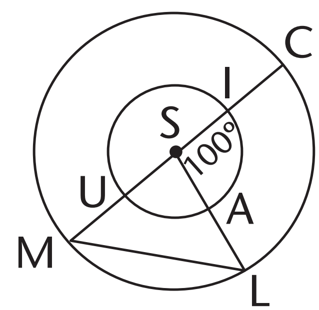
3. Since $\angle USA$ is supplementary to $\angle ASI$, and we can see from the diagram that $\angle ASI$ = 100°, it must be true that $\angle USA$ = 80° (since they must add up to 180°). And the measure of $\overset{\frown}{ML}$ (in other words, $m\overset{\frown}{ML}$), is defined to be the same as the measure of its central angle, which we now know is 80°. Done!

Answer: Both measure 80°.

4. A major arc is one that is larger than 180°, so looking at the smaller circle and starting with the point A, let's move clockwise and we can see that $\overset{\frown}{AUI}$ is a major arc! Its measure would be everything except that 100°, right? So it measures 360° – 100° = **260°**. Doing the same thing but moving counterclockwise, we get the other major arc starting with "A," which is $\overset{\frown}{AIU}$. And what does it measure? Well, it's everything except $\overset{\frown}{AU}$ whose central angle measures 80° (we learned this in #3).

That means $\overset{\frown}{AIU}$ = 360° – 80° = **280°**.

Answer: $\boldsymbol{\overset{\frown}{AUI}}$ **= 260° or** $\boldsymbol{\overset{\frown}{AIU}}$ **= 280°**



5. True! In fact, those are actually two names for the exact SAME angle. ☺

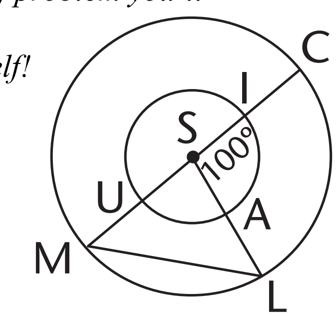
Answer: True

6. False! While it is true that their measures are equal (because their central angle is the same), those arcs have different lengths, so they are not congruent. Remember, in order for two shapes (or arcs) to be congruent, they must be identical in every measurable way – angles and lengths. **Answer: False**

7. The three radii marked on the bigger circle are the three lengths that extend from the center point to the edge of the bigger circle: $\overline{SM}, \overline{SL}, \overline{SC}$.

Answer: $\boldsymbol{\overline{SM}, \overline{SL}, \overline{SC}}$

8. What is the measure of $\angle M$? Well, let's notice that because $\overline{SM}$ and $\overline{SL}$ are both radii of the bigger circle, they must be equal in length, which means that $\triangle MSL$ is an isosceles triangle with base angles that must be congruent: $\angle M \cong \angle L$. Since we know that the third angle in that triangle measures 80° (see problem #3), that means the base angles must add up to 100°, right? (Since all the angles in a triangle must add up to 180°.) And that means each base angle is 50°. Phew, done! *(That's the kind of problem you'll get better at with practice – don't worry if you couldn't do it by yourself! Wanna be superstar? Try it again tomorrow without looking at this solution!)*



Answer: $\boldsymbol{\angle M = 50°}$

9. We know that the measure of $\widehat{LC}$ is 100°, and to find the fraction of the entire circle that the arc takes up, we can just divide this by 360°: $\frac{100°}{360°} = \frac{5}{18}$.[1]

Answer: $\boldsymbol{\frac{5}{18}}$

[1] Advanced footnote: Those degree symbols stand for "degrees", which is a unit, and that unit *cancels away* in the fraction – which is why they disappeared. Mastery of units is really helpful! I do lots of unit/fraction stuff in *Hot X: Algebra Exposed*.

10. If the total circumference of the bigger circle is 24 ft, then we can use the above fraction to find the actual length of $\widehat{LC}$:

Arc length of $\widehat{LC}$ = $\widehat{LC}$'s fraction of circle $\times$ total circumference

$$\text{Arc length} = \frac{5}{18} \times 24 = \frac{120}{18} = \frac{20}{3} = 6\frac{2}{3} \text{ feet}$$

And how many inches is $\frac{2}{3}$ ft? Let's use unit conversions (see chapter 19 in "Math Doesn't Suck" to review this!)

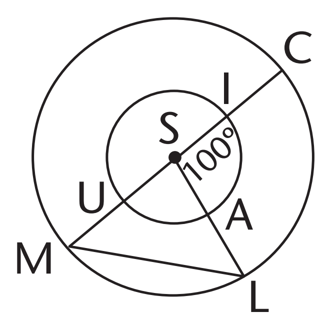
$$\frac{2}{3}\text{feet} \times \frac{12\ inches}{1\ foot} = \frac{24}{3} = 8 \text{ inches}$$

Answer: $6\frac{2}{3}$ feet or 6 feet, 8 inches

11. First of all, what fraction of the smaller circle is $\widehat{UA}$? We know its measure is 80°, so that would be: $\frac{80}{360} = \boldsymbol{\frac{2}{9}}$. And we know this is true from p. 326:

Arc length of $\widehat{UA}$ = $\widehat{UA}$'s fraction of circle $\times$ total circumference

Since we've been given the length of $\widehat{UA}$ = 2, we can solve this! Let's call the total circumference "c."



$$2 = \frac{2}{9} \times c$$

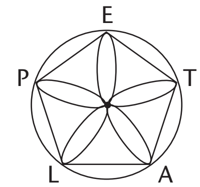
(multiply both sides by 9)

$$18 = 2c$$

$$c = \mathbf{9}$$

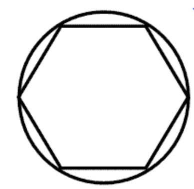
Answer: The total circumference of the smaller circle is 9 feet

12. If the 5 points P, E, T, A & L are evenly distributed around the circle, that means that each arc created will be $\frac{1}{5}$ of the total circumference, since there are 5 arcs of equal length that make it up. Does that make sense? So then the measure of each arc would be $\frac{1}{5}$ the total number of degrees of the circle (360°), which is $\frac{360°}{5}$ = **72°**.



Answer: $\mathbf{\frac{1}{5}}$**; 72°**

13. Here's what a regular hexagon inscribed in a circle looks like!



It's very similar to the PETAL diagram, except there are 6 equal arcs instead of five. And that means each arc is $\frac{1}{6}$ of the total measure, so we can figure out each arc's measure like this: $\frac{360°}{6}$ = **60°**.

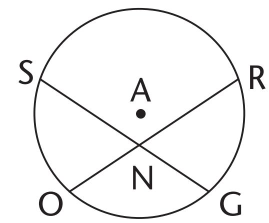
Answer: 60°

14. This time, we're asked about a regular n-gon. We can't really draw this one, but we don't need to! In the previous two problems, we dealt with a regular 5-gon and a regular 6-gon, and for those, we found the measure of a single arc with $\frac{360°}{5}$ and $\frac{360°}{6}$.

Following that pattern, the measure of $\frac{1}{n}$ of a circle would be $\frac{360°}{n}$. That's all there is to it! **Answer:** $\mathbf{\frac{360°}{n}}$

15. In the SARONG diagram, given: $\odot A$, $\overline{SG} \cong \overline{RO}$.

Prove: $\overset{\frown}{SO} \cong \overset{\frown}{RG}$.

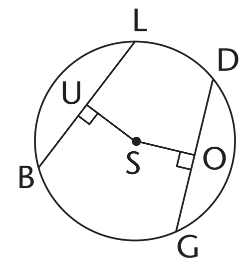


Since we are given $\overline{SG} \cong \overline{RO}$, the Angle-Arc-Chord congruent Rules from p. 328 tell us that $\overset{\frown}{SG} \cong \overset{\frown}{RO}$, right? But we want to prove $\overset{\frown}{SO} \cong \overset{\frown}{RG}$. Hey, the Subtraction Property can help! If we subtract that middle arc, $\overset{\frown}{OG}$, from both of those overlapping congruent arcs, we end up with two arcs that must be congruent: $\overset{\frown}{SO} \cong \overset{\frown}{RG}$. That's it!

♥Proof♥

Statements	**Reasons**
1. $\overline{SG} \cong \overline{RO}$	1. Given
2. $\overset{\frown}{SG} \cong \overset{\frown}{RO}$	2. If two chords are $\cong$, then their corresponding arcs are $\cong$.
3. $\therefore \overset{\frown}{SO} \cong \overset{\frown}{RG}$	3. Subtraction property (subtracting the arc $\overset{\frown}{OG}$ from both $\overset{\frown}{SG}$ & $\overset{\frown}{RO}$)

16. In the BULDOGS diagram, given $\odot S$, $\overline{US} \perp \overline{BL}$, $\overline{SO} \perp \overline{DG}$, $\overset{\frown}{BL} \cong \overset{\frown}{GD}$. Prove: $\overline{US} \cong \overline{SO}$. *(Hint: Remember the Puppies from chapter 18?)*

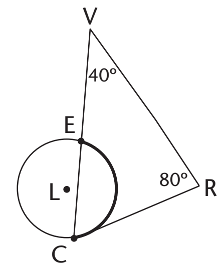


Notice that $\overline{US}$ & $\overline{SO}$ are the <u>distances</u> from the center of the circle to the two chords, $\overline{BL}$ & $\overline{DG}$ (the $\perp$'s in the Givens tell us that). So we are being asked to prove that the chords are equidistant to the center of the circle. Sounds like the Twin Puppies! The Converse of the Twin Puppy theorem from p. 308 says that if the two chords are congruent ($\overline{BL} \cong \overline{DG}$), then the chords must be equidistant to the center **(**$\overline{US} \cong \overline{SO}$). So our new goal is to prove that $\overline{BL} \cong \overline{DG}$. Can we do it? Yes we can! We're given that $\overset{\frown}{BL} \cong \overset{\frown}{GD}$, and the Arc-Angle-Chord Congruent Rules tell us that the chords corresponding to congruent arcs must be congruent. Ta-da! Let's write it out:

♥Proof♥

<u>Statements</u>	**<u>Reasons</u>**
1. $\odot S$, $\overline{US} \perp \overline{BL}$, $\overline{SO} \perp \overline{DG}$, $\overset{\frown}{BL} \cong \overset{\frown}{GD}$	1. Given (The $\perp$ stuff tells us we have *distances* from the center to the chords.)
2. $\overline{BL} \cong \overline{DG}$	2. If two arcs are $\cong$, then their corresponding chords are $\cong$.
3. $\therefore \overline{US} \cong \overline{SO}$	3. Converse of Congruent-chord (Twin Puppy Dog) theorem: If two chords are $\cong$ then the two chords are equidistant to the center.

DTM from p. 337-338



2. In CLEVR, Given: $\odot L$ is tangent to $\overline{CR}$ at the point C, $\angle V$ = 40°, $\angle R$ = 80°. Find $\overset{\frown}{CE}$. (Hint: first, find $\angle C$.)

Let's take the hint! Since the angles in a triangle always add up to 180°, and we're already given two angles in our big triangle (40° + 80° = 120°), that means $\angle C$ = 60°. Ok, what does the question want? It wants us to find $\overset{\frown}{CE}$. Since we know $\angle C$, which is the angle intercepted by that arc, we should focus on figuring out where the armpit is, and go from there! In this case, the armpit (vertex) of the angle is the point "*C*", and now we ask: Is the vertex Peaceful, Lazy, Reckless, or Outcast? Since *C* is on the edge of the circle $\odot L$, that means it's "living on the edge" – reckless! That kind of behavior can cut your lifespan in *half*, so that's how we remember which formula from p. 335 to use. Reckless: $\angle = \frac{1}{2} arc$. Plugging in what we know, this becomes:

$$\angle = \frac{1}{2} arc$$

$$60° = \frac{1}{2}\overset{\frown}{CE}$$

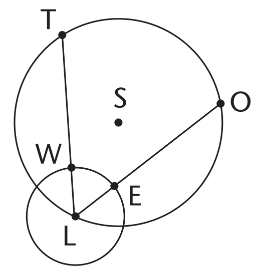
(multiply both sides by 2)

$$\rightarrow 120° = \overset{\frown}{CE}$$

Ta-da!

Answer: $\boldsymbol{\overset{\frown}{CE}}$ = 120°

3. In the TOWELS diagram, Given $\odot S$ & $\odot L$, $\overset{\frown}{TO} = 132°$, and L lies on the circumference of $\odot S$. Find $\overset{\frown}{WE}$.



First of all, let's notice that $\overset{\frown}{WE}$ is intercepted by the angle $\angle L$, and the vertex of $\angle L$ is the point L. What "personality" type is L? With respect to the smaller circle, $\odot L$, the vertex L *peaceful*, because it's at the center of $\odot L$. But with respect to the larger circle, $\odot S$, the vertex L is actually *reckless*, because it's on the edge of $\odot S$. Wait, what is the problem asking for again? It wants to know the measure of $\overset{\frown}{WE}$. That's an arc that's part of the *smaller* circle. However, we would need to know $\angle L$ in order to use one of our angle-arc formulas to find $\overset{\frown}{WE}$. How can we find $\angle L$? Well, what else do we know? Oh, we know that $\overset{\frown}{TO} = 132°$, and that's an arc on the *larger* circle (where L is considered to be reckless) so we can use the reckless formula and $\overset{\frown}{TO}$ to find $\angle L$:

$$\angle = \frac{1}{2} arc$$

$$\angle L = \frac{1}{2}(132°)$$

$$\rightarrow \angle L = 66°$$

Great progress! Now that we know $\angle L$, we can turn our attention to the smaller circle, and use it to find the $\overset{\frown}{WE}$. Which angle-arc formula should we use this time? Well, looking at just the smaller circle, we notice that L is at the center of the circle, so we should use the "peaceful, balanced" formula:

$$\angle = arc$$

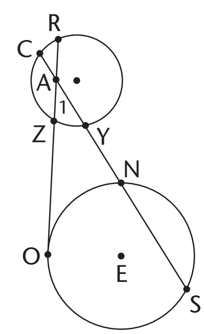
$$\angle L = \widehat{WE}$$

$$\rightarrow 66° = \widehat{WE}$$

Ta-da!

Answer: $\boldsymbol{\widehat{WE}}$ **= 66°**

4. Given: $\overline{RO}$ is tangent to $\odot E$, $\widehat{ON} = 47°$, $\widehat{OS} = 99°$, and $\widehat{CR} = 25\frac{1}{2}°$.



Find $\angle 1$ and $\widehat{ZY}$.

Hm, let's start with what we're given and see what we can figure out. Well, since we know $\widehat{ON}$ and $\widehat{OS}$, we can look at just the bigger circle and the point *A* (which is totally an outcast with respect to the bigger circle!). We pretend nothing in the world exists except the bigger circle and a lone, outcast point "*A*" (we are pretending the small circle doesn't exist right now.) And now we plug what we know into the "outcast" angle-arc formula, and find out $\angle 1$:

$$\angle = \frac{1}{2}(difference\ of\ two\ arcs)$$

$$\angle 1 = \frac{1}{2}(\widehat{OS} - \widehat{ON})$$

(remember, we always subtract the smaller arc's measurement from the bigger one!)

$$\rightarrow \angle 1 = \frac{1}{2}(99° - 47°)$$

$$\rightarrow \angle 1 = \frac{1}{2} \cdot 52° = \mathbf{26°}$$

Great! And now that we know $\angle 1$, we can use it to find $\overparen{CR}$. How? Time to ignore the bigger circle and focus on the smaller circle! With respect to the smaller circle, *A* is floating aimlessly inside the circle, right? That's the "lazy" formula (you know, where people just lead sort of "average" lives…)

$$\angle = \frac{1}{2}(\text{sum of two arcs})$$

$$\angle 1 = \frac{1}{2}(\overparen{CR} + \overparen{ZY})$$

$$\rightarrow 26 = \frac{1}{2}(25\frac{1}{2}° + \overparen{ZY})$$

Not gonna panic! Let's get this thing under control by first multiplying both sides by 2:

$$\rightarrow 52 = 25\frac{1}{2}° + \overparen{ZY}$$

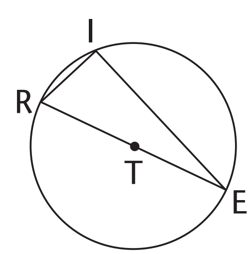
Um, a little better… Now we'll do some careful subtraction, to get:

$$\rightarrow \mathbf{26\frac{1}{2}°} = \overparen{ZY}$$

Phew! Done!

Answer: $\angle \mathbf{1} = \mathbf{26°};\ \overparen{\boldsymbol{ZY}} = \mathbf{26\frac{1}{2}°}$

5. In RITE, Given: $\overline{RE}$ is the diameter of $\odot T$. Find $\angle I$.



(Hint: How many degrees of the circle is $\angle I$ intercepting? And what "personality type" is the point *I*?)

Hm, one of those problems that doesn't seem to have enough information in it. But wait! We are told that $\overline{RE}$ is the diameter of the circle. Notice that means the arc $\overparen{RE}$ must be

180°! Great! Now, we can look at *I*, the vertex of $\angle I$, as a "reckless" vertex, intercepting an arc measuring 180°. *(It might help to imagine erasing the segment $\overline{RE}$.)* Next we plug this info into the "reckless" angle-arc formula (you know, the kind of lifestyle that can cut your life in half!)

$$\angle = \frac{1}{2} arc$$

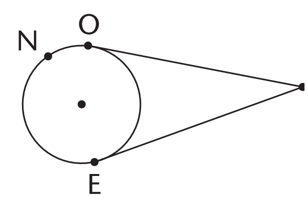
$$\angle I = \frac{1}{2}\overset{\frown}{RE}$$

$$\rightarrow \angle I = \frac{1}{2}\cdot 180°$$

$$\rightarrow \angle I = 90°$$

Ta-da! And it sure looks like a "rite" angle, doesn't it? ☺

Answer: $\angle \boldsymbol{I} = 90°$



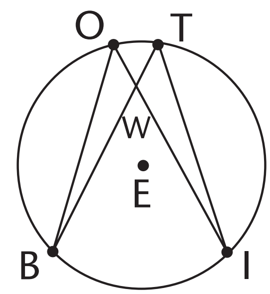
6. In NOPE, given tangent segments $\overline{PO}$ & $\overline{PE}$. Explain why $\overline{OE}$ can't be a diameter in a (paragraph-style) indirect proof. *(Hint: Assume $\overline{OE}$ is a diameter. What would that mean for $\overset{\frown}{OE}$? For $\angle P$?)*

Indirect Proof: Because the vertex of $\angle P$ is outside the circle (an "outcast"), this is true:

$\angle P = \frac{1}{2}\left(\overset{\frown}{ENO} - \overset{\frown}{OE}\right)$, right? Okay, let's assume $\overline{OE}$ is a diameter. Then $\overset{\frown}{OE}$ would equal 180°, and so would $\overset{\frown}{ENO}$. So that would give us: $\angle P = \frac{1}{2}(180° - 180°) = 0$. And $\angle P$ can't be zero! So this is a contradiction, meaning our assumption must have been false. Nice.

$\therefore \overline{\boldsymbol{OE}}$ can't be a diameter.

7. In the BOWTIE diagram, given $\odot E$, $\widehat{BO} \cong \widehat{IT}$, and $\widehat{OT} = 32°$.



Do a paragraph proof to demonstrate that $\overline{OW} \cong \overline{TW}$, using congruent triangles and CPCTC.

Proof: Here's our strategy - we'll use SAA to prove that $\triangle BOW \cong \triangle ITW$ (those two skinny triangles), and then CPCTC will tell us that $\overline{OW} \cong \overline{TW}$. Let's do it! Vertical angles gives us $\angle BWO \cong \angle IWT$ (Gimmie an "A"!). We are given $\widehat{BO} \cong \widehat{IT}$, and because congruent arcs' corresponding chords must also be congruent, we know that $\overline{BO} \cong \overline{TI}$. (Gimmie an "S"!). Since $\angle B$ intercepts $\widehat{OT}$ with *B* on the *edge* of the circle, we know $\angle B = \frac{1}{2}\widehat{OT}$ → $\angle B = \frac{1}{2}(32°)$ = 16°. We could do the same thing with $\angle I$, since *I* is also on the edge of the circle and intercepts the same arc, $\widehat{OT}$. That means $\angle B$ & $\angle I$ both measure 16°, and so $\angle B \cong \angle I$. (Gimmie an "A!"). By SAA, $\triangle BOW \cong \triangle ITW$, and then by CPCTC, $\overline{OW} \cong \overline{TW}$. Phew, done!

$\therefore \boldsymbol{\overline{OW} \cong \overline{TW}}$

Great job reading through these, and hopefully following most of it! You rock!