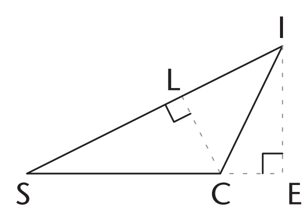
Girls Get Curves

Solution Guide for Chapter 20

Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

DTM from p. 351-352



2. In the diagram SLICE, $\overline{LC}$ and $\overline{IE}$ are altitudes of the triangle $\triangle SCI$. If *SI* = 12 mm, *SC* = 9 mm, and *LC* = 3 mm, find the area of $\triangle SCI$ and the length of $\overline{IE}$.

If we imagine rotating this triangle so that the longest side is flat on the ground, it's a little easier to see how we can apply the good 'ol triangle area formula, using *SI* as the base and *LC* as the height. So we get:

$$\text{Area} = \frac{1}{2}bh$$

$$\text{Area of } \triangle SCI = \frac{1}{2}(SI)(LC)$$

$$\text{Area of } \triangle SCI = \frac{1}{2} \cdot 12 \cdot 3 = \mathbf{18}$$

Great! Now, looking at the triangle exactly as it appears, notice that it's perfectly acceptable to use the base *SC* and altitude *IE* for the area formula, too. And if we plug in what we know, we can solve for *IE*!

$$\text{Area} = \frac{1}{2}bh$$

$$\text{Area of } \triangle SCI = \frac{1}{2}(SC)(IE)$$

$$18 = \frac{1}{2} \cdot 9 \cdot IE$$

(multiplying both sides by 2 to get rid of the fraction)

$$\rightarrow 36 = 9 \cdot IE$$

$$\rightarrow IE = \mathbf{4}$$

Done!

Answer: Area of $\triangle SCI$ = 18 mm^2; *IE* = 4 mm

3. A rectangular pizza's dimensions are 4 inches by 16 inches. A circular pizza has the same area; what must its radius be?

Firstly, the area of the rectangular pizza is just: $bh = 4 \cdot 16 =$ **64**, right?

And the area of the circular pizza would just be given by: Area of circular pizza = πr^2 , right?

Plugging in what we know, and solving for *r*, this becomes:

$$64 = \pi r^2$$

(Now we need to isolate *r*, so let's start by dividing by π.)

$$\rightarrow \frac{64}{\pi} = r^2$$

$$\rightarrow r = \sqrt{\frac{64}{\pi}}$$

$$= \frac{\sqrt{64}}{\sqrt{\pi}} = \frac{8}{\sqrt{\pi}}$$

(To review square roots rules, check out chapter 19 in *Hot X: Algebra Exposed*.) Technically, we should always rationalize the denominator before writing our final answer – this means getting rid of the square root sign in the denominator! How do we do that? Let's multiply our answer times the copycat fraction $\frac{\sqrt{\pi}}{\sqrt{\pi}}$, which of course equals 1, so we're not changing the value of the answer: $\frac{8}{\sqrt{\pi}} = \frac{8}{\sqrt{\pi}} \cdot \frac{\sqrt{\pi}}{\sqrt{\pi}} = \mathbf{\frac{8\sqrt{\pi}}{\pi}}$.

(See p. 186 in Girls Get Curves *to review Rationalizing the Denominator.)*

I know this seems more complicated than $\frac{8}{\sqrt{\pi}}$, but believe it or not, it's considered "simplified form!"

Answer: $\mathbf{\frac{8\sqrt{\pi}}{\pi}}$ **in.**$^{\mathbf{2}}$

4. The radius of Pizza A is three times the radius of Pizza B. What is the ratio of the areas?

Let's assign some variables! Let's call the radius of Pizza A "*a*" and the radius of Pizza B "*b*." Then the area of Pizza A would be: πa^2, and the area of Pizza B would be πb^2, right? So their ratio could be expressed like this:

$$\text{Ratio of Areas: } \frac{\textit{Area of Pizza A}}{\textit{Area of Pizza B}} = \frac{\pi a^2}{\pi b^2}$$

To simplify the ratio, we can certainly cancel a factor of π from top and bottom, and this becomes: $\frac{a^2}{b^2}$, but is there more we can do? Yes! We are told that the radius of Pizza A is three times the radius of Pizza B, in other words, that $\boldsymbol{a = 3b}$. (Think about that for a second – yep, *a* should be bigger than *b*, so we got the order right!) Now we can substitute 3*b* wherever we see *a*, and our ratio becomes:

$$\frac{a^2}{b^2} = \frac{(3b)^2}{b^2} = \frac{3^2 \cdot b^2}{b^2} = \mathbf{\frac{9}{1}}$$

Great, the b^2's cancel away! And that means that no matter how big the pizzas are, if one pizza's radius is three times as big as another, the area will be 9 times as big – always.

Answer: 9 : 1 (or $\mathbf{\frac{9}{1}}$, either one works!)

5. If a 14-inch diameter pizza is cut into 10 equal slices, how much area is *each slice*?

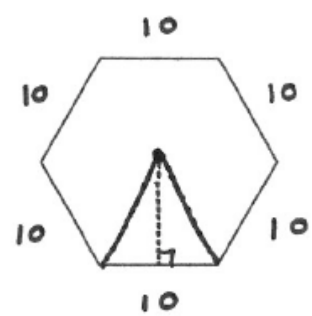
Well, the total area would be πr^2, and since the diameter is 14 inches, that means the radius is 7 inches, right? So the total area of the pizza is just: $\pi r^2 = \pi(7)^2 = 49\pi$.

And if we cut this pizza into 10 equal slices, each slice would be $\frac{1}{10}$ of the total area.

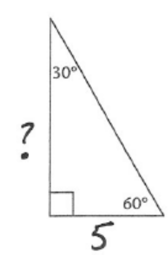
Each slice's area = $49\pi \div 10 = \mathbf{4.9\pi}$. Not so bad, right?

Answer: $\mathbf{4.9\pi}$ in^2

6. Find the area of a regular hexagon with a perimeter of 60 feet. *(Hint: draw a picture. What is the length of each side? Drop an altitude and fill in the rest of the 30°-60°-90° triangle you create.)*



Hm, okay – if the entire perimeter is 60 feet, and it has 6 equal sides, then each side is 10 feet long, right? Let's draw this! This is a regular hexagon, so notice that when we drop an altitude and draw lines to neighboring vertices, we've created an equilateral triangle, or two 30°-60°-90° triangles. (For more on this, check out p. 350 in Girls Get Curves!) Now, if we can find out the area of one of these skinny 30°-60°-90° triangles, then we'd just need to multiply it by 12 to get the area of the entire hexagon, do you see why? And what is the area of one little triangle? We know the base is half the length of a side of a hexagon, which is **5**. And what's the height? Well, since it's a 30°-60°-90° triangle, we can totally find it. (See chapter 11 in Girls Get Curves to review these special traingles!)



In fact, the height will be the shortest side times $\sqrt{3}$! So the height is $5\sqrt{3}$ and our skinny triangle area is:

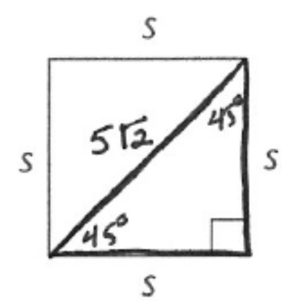
$$\text{Area of skinny triangle} = \frac{1}{2}bh = \frac{1}{2}\cdot 5\cdot 5\sqrt{3} = \frac{25\sqrt{3}}{2}$$

Great! And since 12 of these add up to make the entire hexagon's area, we can find the total area like this: $\frac{25\sqrt{3}}{2} \times 12 = \frac{12\cdot 25\sqrt{3}}{2} = \frac{12\cdot 25\sqrt{3}}{2} = 6\cdot 25\sqrt{3} = \mathbf{150\sqrt{3}}$ **ft²**

Answer: $\mathbf{150\sqrt{3}}$ ft²

7. The diagonals of a square measure $5\sqrt{2}$ cm each. What is the area of the square?

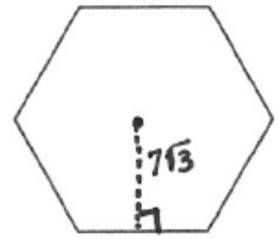
(Hint: use 45°-45°-90° triangles to find the square's sides)



If we had the length of the sides of the square, we could find the area, no problem – it would just be s^2. Let's draw a square with a diagonal drawn in, and we'll see that we have two 45°-45°-90° triangles. Looking at just one of them, if the hypotenuse is $5\sqrt{2}$ cm, that means each of its other sides must equal 5 cm! (Check out chapter 11 for more on that.) And with sides equal to 5 cm, the area of the square is just 5^2 = **25 cm²**.

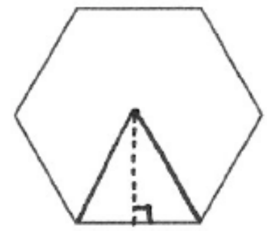
Answer: Area of square = 25 cm²

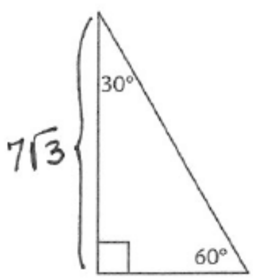
8. If the distance from the center of a regular hexagon to one of its sides is $7\sqrt{3}$ miles, then what are the perimeter and the area of the hexagon?



(Hint: draw a picture!)

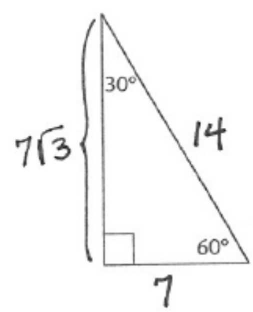
Ok, let's draw a picture! Any time we're dealing with a *distance* from a point to a side, we know that that length creates a *right angle* with the side, so we can draw that in. In order to find the perimeter and the area, we need the length of a side of this hexagon! How can we get it? Since it's a regular hexagon, as we did in #6, we can draw in segments from the center to two neighboring vertices and create an equilateral triangle – which splits into two 30°-60°-90° triangles! Let's do it:





And now, looking at just one of those 30°-60°-90° triangles, since we know the length of the longer leg, we can find the shorter leg by dividing that length by $\sqrt{3}$, which is just:

$\frac{7\sqrt{3}}{\sqrt{3}}$ = **7**. Ah, so nice when that happens!



And in a 30°-60°-90° triangle, the hypotenuse is twice the length of the shorter leg, which is $7 \cdot 2$ = **14**. Nice! Notice that this is also the length of each *side* of the hexagon.

Now, if one side of the regular hexagon is 14 miles, then the entire perimeter must be six times that length, right? That's $14 \cdot 6$ = **84 miles**.

And for the area? We know that the total area is 12 times the area of one of the skinny triangles – so let's find the area of a skinny triangle's area:

$$\text{Skinny triangle area} = \frac{1}{2}bh = \frac{1}{2} \cdot 7 \cdot 7\sqrt{3} = \frac{49\sqrt{3}}{2}$$

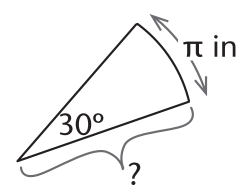
Now we'll multiply that by 12 and get the total area of the hexagon:

$$\text{Area of hexagon} = \frac{49\sqrt{3}}{2} \times 12 = \frac{12 \cdot 49\sqrt{3}}{2} = 6 \cdot 49\sqrt{3} = \mathbf{294\sqrt{3}}$$

Answer: perimeter = 84 miles; area = $\mathbf{294\sqrt{3}}$ **mi**^{2}

(Tip: to multiply 6 times 49 in your head, first multiply 6 times 50 and get 300. Then notice that you have one extra 6 – we only wanted 49 sixes, not 50 sixes – and subtract that 6 back off and get 300 – 6 = 294!)

9. The arc length of the crust on a slice of pizza is π inches. If a protractor tells us the angle of this slice is 30°, what is the length of each side of the slice? *(Hint: This is the same as the radius. Use the arc length to find the diameter and then the radius.)*



Okay, any time we're dealing with a slice of a circle, we should probably go ahead and find the fraction of the pizza that the slice is. Here, since the arc of the crust is 30°, the total fraction of the pizza represented by this slice is: $\frac{30°}{360°} = \frac{3}{36} = \frac{1}{12}$. Great! Now, if the arc length of the crust is π inches, we can figure out the entire circumference:

$$\text{Arc length} = \text{fraction of circle} \times \text{entire circumference}$$

$$\pi = \frac{1}{12} \times C$$

$$\rightarrow 12\pi = C$$

Great! Now we know the entire circumference is 12π inches. How can we use that to find the diameter? We'll use our good 'ol circumference formula, and fill in 12π for C:

$$C = \pi d$$

$$\rightarrow 12\pi = \pi d$$

$$\rightarrow d = 12$$

And that means the radius is 6 inches, which is what the problem was asking for!

Answer: 6 inches

10. The arc length on a (different) slice of pizza is π inches. If a protractor tells us the angle of this slice is 60°, what was the areas of the entire pizza? What is the area of just this slice? (Draw your own picture this time!)

Again, the first thing we do? Figure out the fraction of the entire pizza that this slice is! Since the angle of this slice is 60°, the fraction would be: $\frac{60°}{360°}$ = $\frac{6}{36} = \frac{1}{6}$. Great! So the slice is $\frac{1}{6}$ of the entire pizza. Since we know the arc length of the slice, we can figure out the total circumference of the pizza:

Arc length = fraction of circle × entire circumference

$$\pi = \frac{1}{6}C$$

(multiply both sides by 6 to get rid of the fraction)

$$\rightarrow 6\pi = C$$

Great, so $C = 6\pi$, which is a little bigger than 18, right? (Always nice to do periodic reality checks – π is a number, after all!)

Since the circumference of the circle is 6π, that means we can find now its diameter:

$$C = \pi d$$

Filling in 6π for *C*, we get:

$$\rightarrow 6\pi = \pi d$$

$$\rightarrow d = 6$$

And if the diameter is 6 inches, then the radius is 3 inches! Now it's easy to find the area of the circle; it's just:

$$A = \pi r^2$$

$$\rightarrow A = \pi(3)^2$$

$$\rightarrow A = \mathbf{9\pi}$$

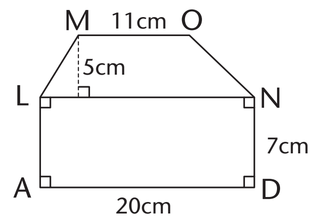
And what is the area of just one slice? It's going to be $\frac{1}{6}$ of the entire area!

$$\text{Area of slice} = \frac{1}{6} \times 9\pi = \frac{9\pi}{6} = \mathbf{\frac{3\pi}{2}}$$

Answer: area of pizza = 9π in^2; area of slice = $\frac{3\pi}{2}$ in^2

DTM from p. 358-359

2. Find the total area of the lopsided house figure, ALMOND. *ALND* is a rectangle, and *LMON* is a trapezoid.



Let's break this down into its parts and then add 'em up – it's a Frankenstein problem!

First, the area of the big lower rectangle is easy:

$$\text{Area of Big Rect} = bh = 20 \cdot 7 = \mathbf{140\ cm^2}$$

And now we have a trapezoid up top. Notice that the trapezoid's bottom side measures 20 cm (feel free to write that in!), so it's a trapezoid with height 5 and bases 11 & 20.

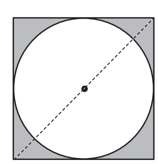
Let's use our trapezoid area formula from p. 353:

$$\text{Area of Trapezoid} = \frac{b_1 + b_2}{2} \cdot h = \frac{(11+20)}{2} \cdot 5 = \frac{31}{2} \cdot 5 = \frac{155}{2} = \mathbf{77\frac{1}{2}\ cm^2}$$

And now we just add 'em up: $\mathbf{140\ cm^2} + \mathbf{77\frac{1}{2}\ cm^2} = \mathbf{217\frac{1}{2}\ cm^2}$

Answer: Total area for ALMOND is $\mathbf{217\frac{1}{2}}$ cm^2

3. For the square/circle diagram to the right, if the diagonal of the square measures $2\sqrt{2}$ cm, what is the shaded area?



Here's our strategy: First we'll find the area of the square, then the area of the circle, and then we'll subtract the circle's area from the square's area – and we'll be left with the area of those curved corners. Let's do it!

Okay, so how do we find the area of the square? We know its diagonals measure $2\sqrt{2}$, so we could notice that the square is divided into two 45°-45°-90° triangles and then realize the sides of the square must equal 2 each, and then get its area like that:

$$\text{Area of Square} = s^2 = 2^2 = \mathbf{4\ cm^2}$$

Or we could use the diagonal (kite) formula on p. 353 – since a square is a kite, after all, and get this:

$$\text{Area of Square} = \frac{1}{2}(d_1 \cdot d_2) = \frac{1}{2}(2\sqrt{2} \cdot 2\sqrt{2})$$

(Rearrange and regroup the terms to make it easier to see!)

$$= \frac{1}{2}([2 \cdot 2] \cdot [\sqrt{2} \cdot \sqrt{2}]) = \frac{1}{2}(4 \cdot 2) = \frac{8}{2} = \mathbf{4\ cm^2}$$

Phew! I prefer the 45°-45°-90° triangle method myself, but I wanted to show you both! Alright, now we need the area of the circle. Do we know the radius? Well, we know the diameter! Notice that the height of the square, 2 cm, is the same as the diameter of the circle. And that means the radius of the circle is half that – in other words, the radius of the circle is 1 cm.

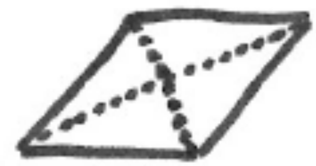
$$\text{Area of circle} = \pi r^2 = \pi(1)^2 = \pi(1) = \boldsymbol{\pi}\ \mathbf{cm^2}$$

So the shaded area is just the area of the square minus the area of the circle: **4 – π**

Answer: Shaded Area = (4 – π) cm^2

…or you could just write 4 – π cm^2. The parentheses are there to make it clear that the units apply to both numbers, not just to the π!

4. A rhombus's area is 25 cm^2, and one of its diagonals is twice the length of the other. What are the lengths of the diagonals? *(Hint: Draw a picture, label the shorter diagonal "x" and the longer one "2x" and use the kite formula on p. 353.)*



Okay, let's sketch a rhombus and draw in the dotted diagonals. It doesn't need to be pretty; just good enough so we can see what's going on! If one of the diagonals has length x and the other has the length $2x$, then let's plug those into the kite area formula we're given on p. 353 (since rhombuses are kites, we can totally use it).

$$\text{Area of Rhombus: } \frac{1}{2}(d_1 \cdot d_2)$$

$$= \frac{1}{2}(x \cdot 2x) = \frac{1}{2}\left(2x^2\right) = x^2$$

And hey, we've been told that the area equals 25, so we can easily solve for x:

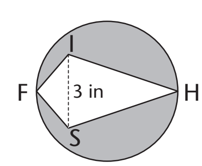
$$25 = x^2$$

$$\rightarrow x = 5$$

Remember: The problem wanted the two diagonals, which we originally labeled x and $2x$, right? And since we discovered $x = 5$, now we know that the diagonals are 5 and 10. Done!

Answer: 5 cm and 10 cm

5. For the FISH diagram to the right, if the short diagonal of the kite, $\overline{IS}$, measures 3 in. and the diameter of the circle, $\overline{FH}$, measures 10 in., then what is the shaded area? *(Hint: This is multistep; keep your brain on!)*



Okay, another subtraction problem! First we'll find the area of the circle, then the kite, and then we'll subtract the kite area from the circle area. What's the circle area? Well if its diameter is 10, then its radius is 5.

$$\text{Area of Circle} = \pi r^2 = \pi(5)^2 = \mathbf{25\pi}.$$

And what's the area of the kite? We have a nifty formula for the area of a kite on p. 353 that uses diagonals, and we've been given the diagonals! Easy-peasy:

$$\text{Area of Kite} = \frac{1}{2}(d_1 \cdot d_2) = \frac{1}{2}(3 \cdot 10) = \frac{30}{2} = 15$$

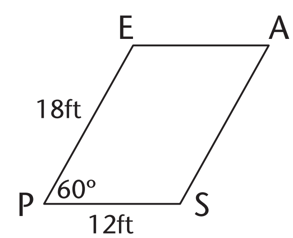
So the shaded area = Area of Circle – Area of Kite = **25π – 15**

And there's no way to simplify it further!

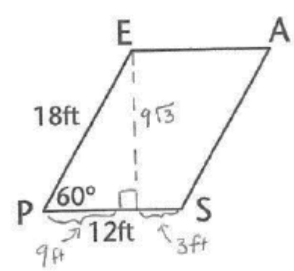
Answer: (25π – 15) in^2

…or you could just write 25π – 15 in^2. The parentheses are there to make it clear that the units apply to both numbers, not just to the 15!

6. Find the area of the parallelogram PEAS. *(Hint: Draw the altitude from the E down to $\overline{PS}$.)*



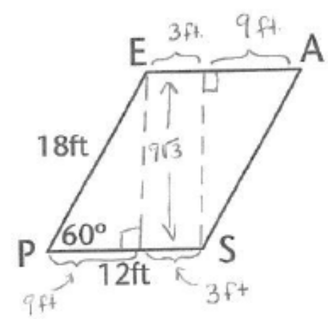
Let's take the hint and draw a segment from *E*, down to $\overline{PS}$, making a right angle with it, which means we've drawn the altitude. What does that do for us? Well, we've created a right triangle – in fact, a 30°-60°-90° triangle! We just drew the 90°, the 60° is already marked, and that means the third angle must measure 30°. We know the hypotenuse is 18 ft, and we know from chapter 11 that that for *all* 30°-60°-90° triangles, the shorter leg must be *half* the hypotenuse. So the



bottom side of our triangle must be 9 ft. Make sense?

And we also know that in all 30°-60°-90° triangles, the longer leg is the shorter leg times $\sqrt{3}$, so that means the altitude we just drew must equal $9\sqrt{3}$. Nice progress!

So, what was this problem wanting us to find? The area! Looks like a Frankenstein problem to me... In fact, let's draw the altitude from *S* up to $\overline{EA}$, and we'll be able to divide this puppy into three areas: two triangles and a skinny rectangle. Since this is a parallelogram, we know the upper right angle, $\angle A$, also equals 60°, and that $\overline{SA}$ must be the same length as $\overline{PE}$. In other words, *SA* = 18. It's the same triangle as we just worked with, but upside down on the other side!



What is the area of one of these triangles?

Area of Triangle = $\frac{1}{2}bh = \frac{1}{2}(9)(9\sqrt{3}) = \frac{81\sqrt{3}}{2}$.

So that means the area of both triangles is $\frac{81\sqrt{3}}{2} \times 2 = \mathbf{81\sqrt{3}}$.

And what is the area of the skinny rectangle? Hm, we know the height is $9\sqrt{3}$, but what is the width? It's just 12 – 9 = **3**. Great! So the area of the skinny rectangle is $3 \times 9\sqrt{3} = \mathbf{27\sqrt{3}}$.

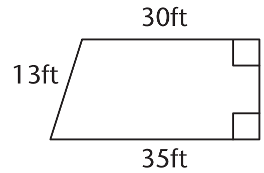
Total area = area of both triangles + area of skinny rectangle

$$= \mathbf{81\sqrt{3}} + \mathbf{27\sqrt{3}} = \mathbf{108\sqrt{3}}$$

Nicely done!

Answer: $\mathbf{108\sqrt{3}}$ **ft^2**

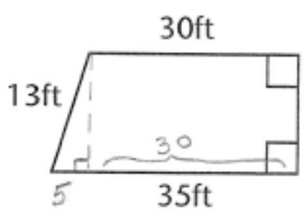
7. Find the area of this trapezoid.

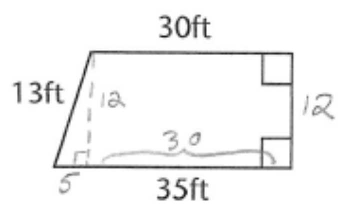


(Hint: We saw this diagram on p. 185.)

If you did #14 on p. 185, then part of this might look familiar! We found the missing side of this trapezoid and also the perimeter. This time we need the missing side, because we need it in order to find the area…

Here's how it works: The missing side of this trapezoid is the same as the height, and if we draw in an altitude from the upper left vertex, that is the height, by definition! Doing that (see below), we've created a right triangle and a rectangle. The rectangle's top and bottom sides must be equal, which means the "leftover" segment on the bottom must equal 5 ft. And that's a *leg* of our right triangle (not necessarily a 30°-60°-90° triangle!)





Now we have two lengths on the right triangle, a leg = 5 and hypotenuse = 13. Well that's a 5-**12**-13 triple, like we saw in Chapter 11! (We also could have used the Pythagorean Theorem to find the 12). That means the altitude we drew must have a length of 12 ft, which means the missing side of the trapezoid is also **12 ft**.

Great! And now we can use our handy-dandy trapezoid formula to find the area.

$$\text{Area of Trapezoid} = \frac{b_1 + b_2}{2} \cdot h = \frac{30+35}{2} \cdot 12 = \frac{65}{2} \cdot 12 = 65 \cdot 6 = \mathbf{390\ ft^2}$$

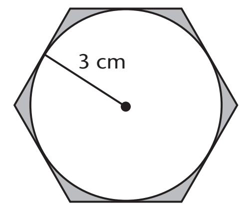
Or in this case, we have enough information to find the area of the triangle and the rectangle and add 'em together. Either one works!

Area of Triangle: $\frac{1}{2}bh = \frac{1}{2}\cdot 5\cdot 12 = \frac{60}{2} = \mathbf{30}$

Area of Rectangle = 30 · 12 = **360**

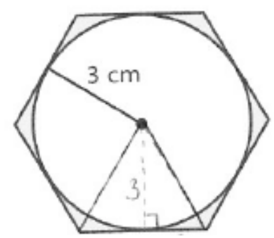
Total Area = **30 + 360** = **390**

Answer: 390 ft^2



8. A circle is drawn inside a regular hexagon so that each side is tangent to the circle. The circle's radius is 3 cm. What is the distance from the center to a vertex of a hexagon? What is the area of the shaded region? *(Hint: This is different from #1! Draw one of the equilateral triangles like we did on p. 350 to see what's going on.)*

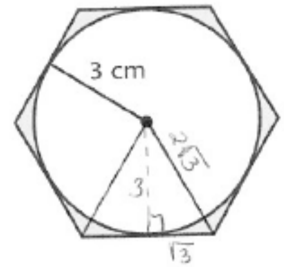
Hm. As we've seen many times in this chapter, since we're dealing with a hexagon, we know that if we draw segments from the center to the vertices, we create 6 equilateral triangles. Let's draw one of them and notice that the height of the equilateral triangle is 3 cm. We have the height of the triangle, but what is its base? Again, as we've seen many times in this chapter, if we look at *half* of any equilateral triangle, we get a 30°-60°-90° triangle, and since the longer leg is 3, that means we can get the *shorter* leg by just dividing by $\sqrt{3}$. In other words, the shorter leg is $\frac{3}{\sqrt{3}}$. Great progress! But before we do anything else, let's rationalize the denominator so that we don't have a radical on the bottom of the fraction (see chapter 11 to refresh on how to do this). We'll multiply it by the copycat fraction $\frac{\sqrt{3}}{\sqrt{3}}$, and we get:



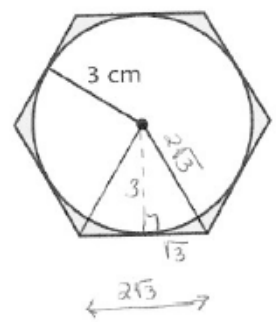
$$\frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}} = \frac{3\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

How about that! By rationalizing the denominator, we actually ended up getting rid of the denominator. Nice.

Now, since the shorter leg is $\sqrt{3}$, and in every 30°-60°-90° triangle, the hypotenuse is *twice* the length of the shorter leg, that means the hypotenuse = $2\times\sqrt{3}$ = $\mathbf{2\sqrt{3}}$. And that's the distance from the center of the circle to a vertex – which is part of what the problem was asking for. Great!



For the second part of the problem, we'll want to find the area of the hexagon, and then subtract the area of the circle from it. In order to find the area of the hexagon, we can find the area of one of the equilateral triangles, and then just multiply it by 6, because there are 6 of these equilateral triangles that make up the hexagon. So we'll need a base and height for the triangle. We already have the height – it's 3. For the base, we can either notice that since it's an equilateral triangle, the base must be the same as its side: $2\sqrt{3}$. Or we could even notice that we had found out half the length of that base earlier, which was $\sqrt{3}$, and if that was half, then the whole base must be $2\sqrt{3}$! Either way gets us the same result, which is nice. ☺



So now to get the area of the equilateral triangle, we can do:

$$\text{Area of equilateral triangle} = \frac{1}{2}bh = \frac{1}{2}(2\sqrt{3})(3) = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$$

And the hexagon is made up of 6 of these triangles, so:

$$\text{Hexagon area} = 6\cdot 3\sqrt{3} = \mathbf{18\sqrt{3}}$$

The area of the circle is easy; we know the radius, so we just do:

$$\text{Area of Circle} = \pi r^2 = \pi \cdot 3^2 = \mathbf{9\pi}$$

And that means the shaded area is given by:

$$\text{Shaded area} = \text{Hexagon Area} - \text{Circle Area} = \mathbf{18\sqrt{3}\ -\ 9\pi}$$

Ta-da!

Answer: Distance = $\mathbf{2\sqrt{3}\ cm}$; **Area =** $\mathbf{18\sqrt{3} - 9\pi\ \ cm^2}$

By the way, we could have done this problem without ever rationalizing the denominator, but the fractions would have been messier to deal with. If you want to see the nuts and bolts of how that would go, see below:

The shorter side of the 30°-60°-90° triangle would have been left as $\frac{3}{\sqrt{3}}$*, and the sides of the triangles would have been twice that:* $\mathbf{\frac{6}{\sqrt{3}}}$*, which is also the distance from the center to a vertex.*

So the area of the equilateral triangle = $\frac{1}{2}bh = \frac{1}{2}(\frac{6}{\sqrt{3}})(3) = \frac{9}{2\sqrt{3}}$.

And that means the total hexagon area $= \frac{9}{2\sqrt{3}} \times 6 = \frac{54}{2\sqrt{3}} = \frac{27}{\sqrt{3}}$.

Which means the total shaded area = $\frac{27}{\sqrt{3}} - 9\pi\ \text{cm}^2$

And the answer would have been (if we didn't rationalize it at the end – which we should!):

Answer: Distance = $\mathbf{\frac{6}{\sqrt{3}}}$ ***cm; Area =*** $\mathbf{\frac{27}{\sqrt{3}} - 9\pi\ cm^2}$

But I sure like the rationalized version more, and so will most teachers!