Girls Get Curves

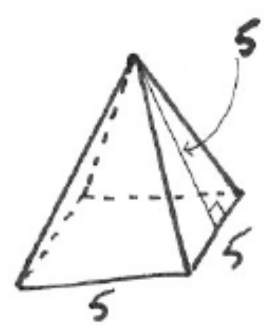
Solution Guide for Chapter 21

Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

DTM from p. 374-375

2. Find the surface area of a pyramid with slant height 5 in, whose Base is a square with sides of length 5 in.

Let's draw it; we start with a square base (but slanted, like if it were laying on the ground) and then draw a dot over it, and then draw lines down to each of the squares' vertices (and use dotted lines for the edges that are supposed to be "unseen") and add in the slant height, which should be laying on the surface of the triangle, making a right angle with the bottom.



Now we can see that the total surface area will be the sum of the Base and each of the 4 triangles. Let's find the Base's area now:

Area of Square Base = $s^2 = 5^2 =$ **25**

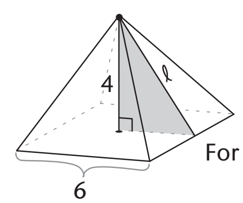
And remember that for surface area of the triangles, we need the *slant height* of the triangles, *not* the height of the pyramid. So instead of $\frac{1}{2}bh$ for the area of the triangles, we'll use: $\frac{1}{2}bl$. Here, the slant height (l) is 5. And the base "b" for the triangles is just the length of a side of a square – which also happens to be 5!

Area of Four Triangles = $4\times\frac{1}{2}bl = 4\times\frac{1}{2}(5)(5) = 4\times\frac{25}{2} = 2\times 25 =$ **50**

For the total surface area, we just add 'em up: **25 + 50 = 75**

Answer: 75 in^2

3. Find the surface area of a pyramid with height 4 meters and whose Base is a square with sides of length 6 meters. See the picture. *(Hint: First use the shaded triangle to find the slant height.* *See p. 372 for an example with a cone.)*



For #3

Let's start with the easy part – finding the surface area of the Base:

Area of Square Base $= s^2 = 6^2 =$ **36**

Okay, now we're given the height of the pyramid, but we'd need the *slant height* to find the surface area of the triangle-shaped sides – and happily, the hint gives us an idea for how to find that slant height! Much like we did on p. 372, we'll look at an internal, right triangle that is made up of the height, the slant height, and half the length of the Base. Half the length of the Base would be 3, see that? So in this case, the triangle we get has sides 4, 3, and a hypotenuse labeled l. So we could use the Pythagorean theorem to find the hypotenuse, or we could just notice that it's a 3-4-5 triangle, which means the hypotenuse is **5**. Nice.

Remember – for the areas of these triangles, we need the height of the triangles (slant height *l*), not the *h*eight of the pyramid!

$$\text{Area of Four Triangle Sides} = 4\times\frac{1}{2}bl = 4\times\frac{1}{2}(6)(5) = 4\times\frac{30}{2} = 2 \times 30 = \mathbf{60}$$

And now we add 'em together!

Total Surface Area = S.A. of Square Base + Lateral S.A. (the triangles)

Total Surface Area = **36 + 60 = 96**

Answer: 96 $\mathbf{m^2}$

4. Find the surface area of a sphere with diameter 10 mm.

The only trick here is to remember that the surface area formula uses the radius, not the diameter! So, using the radius 5 mm, we get:

$$\text{Surface Area of a Sphere} = 4\pi r^2 = 4\pi(5)^2 = 4\pi(25) = \mathbf{100\pi}$$

Answer: 100π $\mathbf{mm^2}$

5. Find the surface area of a right prism with height 7 inches, whose Base has a total perimeter of 8 inches, and each Base area is 4 square inches.

Hm, no picture! What does each Base look like? Who knows? They could be squares, rectangles, or some crazy weird polygons – we just don't know. But we *do* know everything we need to find the surface area. The Base areas are given to us, so that part's done. How about the lateral surface area? Imagine a random polygon (octagon? 120-gon?) Base and the sides extending up from them – how do we find its surface area?

Actually it's not so hard: Imagine cutting open a toilet paper core open so that we "unfold" it into a big rectangle, whose width is indeed the perimeter of the Base (the circumference, in that case!). Now imagine the toilet paper core again, as a cylinder, but imagine if we bent the toilet paper core with our hands so that the "Base" became a square – or a rectangle, or really, any shape at all! No matter how weird the base of a right prism might be, we can always cut it open with a straight line (that is perpendicular to the edge of the Bases) and it WILL unfold into a big rectangle, whose width is the same as the perimeter of the Base. Read that a few times if you need to! It's great practice to visualize this stuff.

So, since we know the height of the prism and the perimeter, we know the lateral surface area – it's just the big rectangle we get from "unfolding" the prism!

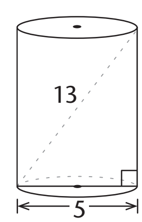
Lateral Surface Area = bh = (entire perimeter)h = (8)(7) = **56**

And we were told the Base Surface areas are just 4 + 4 = **8**, so the entire Surface area is:

Surface Area = Base Areas + Lateral Area = **8 + 56 = 64**

Answer: 64 in^2

6. Find the surface area of the cylinder to the right, whose Base has a diameter of 5 meters and a diagonal (as depicted) of 13 meters. *(Hint: First find the height of the cylinder.)*



I see a right triangle! Do you see it? In fact, I see a 5-12-13 right triangle with the "12" not labeled. You see, because of where the right angle is, we know the 13 is the hypotenuse – and with one leg being 5, that *forces* the other leg to be 12. Notice that the height of the cylinder equals the side of the triangle – which we now know is 12. (Check out chapter 11 to review 5-12-13 triangles.) So, ***h* = 12**. Great progress! For the lateral surface area, we'll need the circumference of the circle Bases, which we can find like this:

$$C = \pi d = \pi(5) = \mathbf{5\pi}$$

Now we have enough info to calculate the entire surface area:

$$\text{Lateral Surface Area} = bh = (5\pi)(12) = \mathbf{60\pi}$$

Time to calculate the area of the Bases. Notice that the radius is *half* of 5, in other words, $r = \frac{5}{2}$.

$$\text{Surface Area of Two Bases} = 2\times\pi r^2 = 2\times\pi\left(\frac{5}{2}\right)^2 = 2\times\pi\left(\frac{25}{4}\right) = \frac{50\pi}{4} = \mathbf{12.5\pi}$$

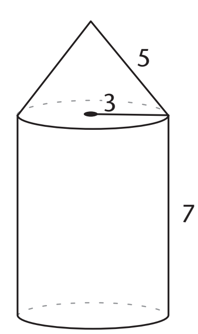
And now we add 'em up:

$$\text{Total Surface Area} = \text{Area of Bases} + \text{Lateral Area} = \mathbf{12.5\pi + 60\pi = 72.5\pi}$$

Done!

Answer: 72.5π $\mathbf{m^2}$

7. Find the surface area of the oversized crayon to the right whose measurements are in feet. Remember, surface area is just the total area we need to cover with wrapping paper! (Hint: Don't blindly use formulas.)



Let's take this in parts! For the cylinder part, we need the bottom Base area and the lateral area – not the top Base, because that part wouldn't have wrapping paper on it, see that? In fact, there's only one Base in this diagram that contributes to surface area! Let's find that now. Remember, the instructions in this chapter said we could assume it's a right cylinder, so that means the Bases are identical – which means the radius of the bottom Base is also 3.

$$\text{Base Surface Area} = \pi r^2 = \pi(3)^2 = \mathbf{9\pi}$$

Next, let's look at the lateral surface area for the cylinder – it's just a big rectangle whose height is 7 and width is the circumference of the Base, which is: $C = \pi d = \pi 6 = 6\pi$. So:

$$\text{Cylinder Lateral Surface Area} = bh = (6\pi)(7) = \mathbf{42\pi}$$

Now we'll tackle the lateral surface area of the cone. Remember the "fan" formula? When we cut open the lateral surface area of a cone, it looks like a "fan"… perhaps made of pirls – I mean, pearls!

$$\text{Cone Lateral Surface Area} = \text{"pirl"} = \pi r l = \pi(3)(5) = \mathbf{15\pi}$$

And now we put 'em all together!

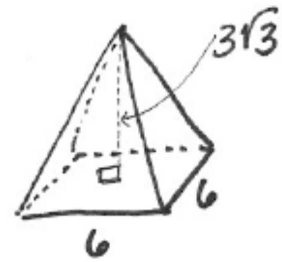
Total Surface Area = Base Area + Cylinder Lateral Area + Cone Lateral Area

$$\text{Total Surface Area} = \mathbf{9\pi + 42\pi + 15\pi = 66\pi}$$

Answer: 66π ft^2

DTM from p. 380-381

2. What is the volume of a pyramid whose Base is a square with sides = 6 cm each and whose height is $3\sqrt{3}$ cm?



Let's draw this! Hm, it looks like a straightforward application of the pyramid volume formula:

$$\text{Pyramid Volume} = V = \frac{1}{3}Bh$$

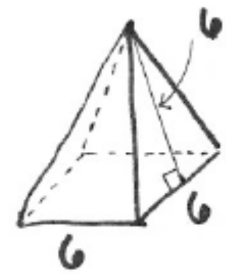
We know that $h = 3\sqrt{3}$. But what is B? It's the area of the Base, which in this case is a square: Area = $s^2 = 6^2 = \mathbf{36}$.

$$\text{Pyramid Volume} = V = \frac{1}{3}Bh = \frac{1}{3}(36)(3\sqrt{3}) = 12(3\sqrt{3}) = \mathbf{36\sqrt{3}}$$

Not so bad, right?

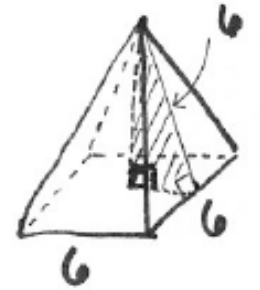
Answer: $\mathbf{36\sqrt{3}}$ cm^3

3. What is the volume of a pyramid whose Base is a square with sides = 6 cm each and whose slant height is 6 cm?

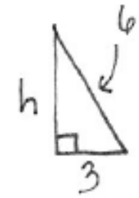


Let's draw a picture again, and notice that we are being given different kinds of information this time. Remember, *slant height* makes a right angle with the base of the triangle side – the slant height does *not* make a right angle with the big square Base (the *height* does that, like in #2 above – make sure you see the difference!).

In order to find the volume of this pyramid, we will need the height of the pyramid – how can we find that? Well, first we should draw in the height – we draw a line from the top of the pyramid down to the center of the square Base, making a right angel with it. And then we notice that there is a right triangle inside the pyramid that we can use! We saw this kind of thing in #3 from p. 374. It's a common trick for these kinds of problems, so it's good to be familiar with the strategy.



Now let's isolate the triangle so we don't get confused about which right angle marker is actually part of the right triangle! Notice that the shorter leg of the right triangle is just half the length of the Base, which is just 3. Now we can use the Pythagorean theorem to find h:



$$a^2 + b^2 = c^2$$

$$h^2 + 3^2 = 6^2$$

$$h^2 + 9 = 36$$

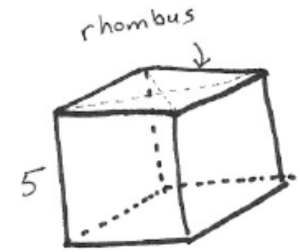
$$h^2 = 27$$

$$h = \sqrt{27} = \sqrt{9 \cdot 3} = 3\sqrt{3}$$

Great! Now we know the height of the pyramid is $3\sqrt{3}$, in other words, $h = 3\sqrt{3}$. And hey, we've discovered that this is the exact same pyramid as in #2! So it's got the same volume, too. Nice!

Answer: $36\sqrt{3}$ cm^3

4. The Bases of a prism are rhombuses with diagonals equal to 4 inches and 6 inches. If the prism is 5 inches tall, what is the volume of the prism? *(Hint: Draw a picture, and see pp. 352-353 for how to find the area of a rhombus.)*



Ok, so we have a prism whose Bases are *rhombuses* – let's draw it, and include its (thin, dotted) diagonals. In order to find the volume of a prism, we need the area of the Base, and then we multiply it by the height. No problem! So we just need the area of one of these rhombuses. And we know from p. 353 that, because a rhombus *is* a kite, we can find the area of a rhombus by using the kite formula – which uses its diagonals!

$$\text{Rhombus Area} = \frac{1}{2}(d_1 \cdot d_2) = \frac{1}{2}(4 \cdot 6) = \frac{24}{2} = 12$$

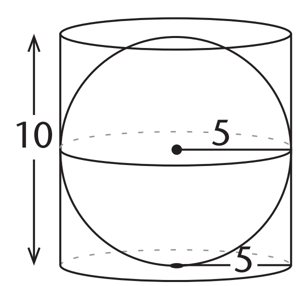
Great! And now, since we're told that the height of the prism is 5:

$$\text{Volume of a prism} = Bh = (12)(5) = \mathbf{60}$$

Done!

Answer: 60 in^3

5. A solid ball of ice cream with radius 5 cm is placed in a small can, which was already filled to the rim with root beer. Of course, root beer spills everywhere. The can is a cylinder with base of radius 5 cm and a height of 10 cm.



Part a. What was the original volume of root beer?

Part b. What is the volume of the ice-cream ball?

Part c. And finally, after the ice cream is put in and root beer spills everywhere, how much root beer is actually still left inside the can?

Ok! First things first. Part a wants the volume of the can, right? That's the same as the volume of the original root beer, after all. No prob!

The area of the Base is just $\pi r^2 = \pi(5)^2 = 25\pi$. And we know that $h = 10$, so we can do:

$$\text{Volume of cylinder} = Bh = (25\pi)(10) = \mathbf{250\pi}$$

Part "a" done!

Next: what is the volume of the ice-cream ball? Again – easy – we just use the "volume of a sphere" formula, with 5 as the radius again. (Remember the birdseed from p. 378!)

$$\text{Volume of Sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5)^3 = \frac{4(125)\pi}{3} = \mathbf{\frac{500\pi}{3}}$$

Part "b" done!

And for part c, it wants the amount of root beer *still left in the can*, once the ice cream was put in. And that's going to just be the original volume of root beer, minus the volume of ice cream! (Because the amount of root beer that spilled out of the can is equal to the volume of ice cream put into the can.)

$$\text{Volume of Remaining Root Beer} = \text{Volume of Can} - \text{Volume of Ice Cream}$$

$$\text{Volume of Remaining Root Beer} = 250\pi - \frac{500\pi}{3}$$

Now let's actually *do* this subtraction. (Remember, we can treat π like a variable – and as long as the "variable parts" of two expressions match, we have "like terms" and we can freely add or subtract them – check out chapter 9 in Kiss My Math for more on this).

How do we combine these two fractions? We need a common denominator! So we'll multiply the 250π times the copycat fraction $\frac{3}{3}$ and our problem becomes:

$$\left(\frac{3}{3}\right)250\pi - \frac{500\pi}{3} \quad = \quad \frac{750\pi}{3} - \frac{500\pi}{3} \quad = \quad \boldsymbol{\frac{250\pi}{3}}$$

Answer:

Part a: Original root beer volume = $\mathbf{250\pi\ cm^3}$

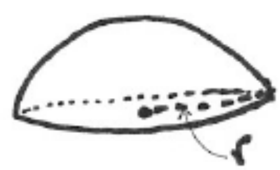
Part b: Ice cream ball volume = $\mathbf{\frac{500\pi}{3}\ cm^3}$

Part c: Remaining root beer volume = $\mathbf{\frac{250\pi}{3}\ cm^3}$

6. A chef wants to shape a small serving of risotto into a perfect, half-sphere-shaped dome.

Part a. If the volume of the risotto is $\frac{16}{3}\pi$ in.^{3}, then what is the radius of the resulting dome? *(Hint: What would be the formula for a half-sphere?)*

Part b. If coconut is to be sprinkled over the risotto, what is the surface area that will be covered with coconut?



Here's a little sketch of a half-sphere dome. Ok, I'm not Michelangelo… anyway, we want to find the radius, and we've been given the volume. We know the volume of a sphere is $\frac{4}{3}\pi r^3$, so the volume of a half-sphere would be: $\frac{1}{2}\times\frac{4}{3}\pi r^3 = \frac{4}{6}\pi r^3 = \frac{2}{3}\pi r^3$. Great! Now we can set that equal to the

volume we've been given $\left(\frac{16}{3}\pi\right)$, and solve for r.

Half-sphere volume:

$$\frac{16}{3}\pi = \frac{2}{3}\pi r^3$$

(multiply both sides by 3)

$$\rightarrow 16\pi = 2\pi r^3$$

(divide both sides by 2)

$$\rightarrow 8\pi = \pi r^3$$

(divide both sides by π)

$$\rightarrow 8 = r^3$$

(take the cube root of both sides)

$$\rightarrow \boldsymbol{r = 2}$$

Great! Part "a" is done. The radius is **2 inches**.

For part b, we need to find the surface area of the dome that would be covered in coconut. Since it's being sprinkled on top, there won't be any coconut on the bottom (circular) face – that part of the risotto will just be touching the plate, after all! So all we need to find is the surface area of the top dome part. And what is the formula for the surface area of a sphere? Surface Area = $4\pi r^2$. In this case, we only need half that area, so:

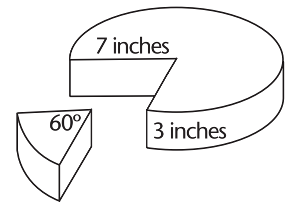
$$\text{Surface Area of Half-Dome} = 2\pi r^2 = 2\pi(2)^2 = 2\pi(4) = \boldsymbol{8\pi}$$

Answer:

Part a. $\boldsymbol{r}$ **= 2 inches**

Part b. Surface area = $\boldsymbol{8\pi}$ **in**$^{\mathbf{2}}$

7. A birthday cake has a radius of 7 inches and a height of 3 inches. A piece of cake is cut, creating a 60° angle. What was the volume of the whole cake before the piece was cut? What is the remaining volume of the cake without this piece?



The volume of the whole cake? Piece a'… nevermind. It's just a squatty cylinder, with radius 7 and height 3. That means the area of the circle Base is $\pi r^2 = \pi(7)^2 = 49\pi$. And so:

$$\text{Volume of Cylinder} = Bh = (49\pi)(3) = \mathbf{147\pi}$$

So that's the volume of the entire cake. First part done!

For the next part, we need the volume of the remaining cake after the piece is cut. To find the volume of the remaining cake, we need to figure out *what fraction of the entire cake is left over* after the piece is taken out. We know the piece's arc measures 60°, which means the arc of the remaining cake measures 360° – 60° = **300°**. See that? Now let's figure out what fraction of the whole thing that is: $\frac{300°}{360°} = \frac{30}{36} = \mathbf{\frac{5}{6}}$. Great! So the volume of the remaining cake is exactly $\frac{5}{6}$ of the total volume:

$$\text{Volume of remaining cake} = \frac{5}{6} \times 147\pi$$

That might seem like a monster fraction to multiply (in your head, anyway!) but we can make it better! Notice that 147's digits add up to a number divisible by three: 1 + 4 + 7 = 12, so by the special "3" divisibility rule (see p. 11 in Math Doesn't Suck for more divisibility tricks!), we know 147 must also be divisible by 3. In fact, we can do that in *our heads and get the factorization: 147 = 3 × 49. And hey, if we look up to the "Volume of a Cylinder" line from the previous page, that's um, exactly how we got 147. ;)*

Moving forward, we can now simplify things:

Volume of remaining cake = (Fraction of the cake is it) × (Total cake volume)

Volume of remaining cake $= \frac{5}{6} \times 147\pi$

(multiplying and reducing it) $= \frac{5 \cdot 3 \cdot 49\pi}{6} = \frac{5 \cdot \cancel{3} \cdot 49\pi}{2 \cdot \cancel{3}} = \frac{5 \cdot 49\pi}{2}$

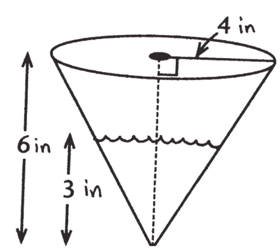
And 49 times 5 isn't so bad – pretend we were multiplying 50 times 5; we'd get 250, right? Now recognize that we have one "extra" 5 hanging around – we only wanted 49 5's, not 50 5's, and subtract 5 off, to get: 245. And we get: $\mathbf{\frac{245\pi}{2}}$. Done!

Answer:

Part a. 147π in^3

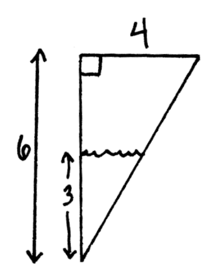
Part b. $\frac{245\pi}{2}$ in^3

8. A cone-shaped cup is 6 inches tall with a radius of 4 inches.



Part a. How much total water could the cup hold?

Part b. If water is poured out until the water is only 3 inches high, what is the new radius that the top of the water makes? What is the new volume of water in the cup? (Hint: See the drawing to the right. Use similar triangles and proportions to find the new radius, and then use that radius to find the new volume!)

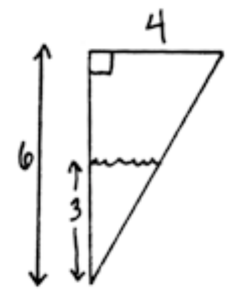


Alright, part "a" is easy – we just need to find the total volume of the cone! Don't let the upside-down nature of the cone fool you – it's still just a cone. ☺ The radius of the circular Base is 4, so that means the area of the Base is: $\pi r^2 = \pi(4)^2 = 16\pi$. And since the total height is 6, we can easily calculate the volume:

$$\text{Volume of Cone} = \frac{1}{3}Bh = \frac{1}{3}(16\pi)(6) = (16\pi)(2) = \mathbf{32\pi}$$

So the total volume of water the cup could hold is **32π in.^{3}**. Part "a" done!

For part b, we pour out water until the water level is only 3 inches high. How can we find the radius that the top of the water makes? Let's take a look at the second diagram – there are two triangles – the big one that is 6 inches tall, and the smaller one made of just the remaining water, which is 3 inches tall.



We can assume the cup is being held upright, and that its rim is parallel to the ground. In that case, notice that the two angles on the right side of the diagram must be congruent – because they are corresponding angles on a transversal (escalator) at a mall with parallel floors. With me so far? Now because of AA~, we know that the two triangles in our diagram are *similar*! In fact, they are similar *right* triangles, so if we wanted to, we could draw in a right angle marker where the water touches the center line, directly below the right angle marker that is already there.

How does it help us if those two right triangles are similar? Because now corresponding sides must be proportional! (See chapter 17 in *Girls Get Curves* to review this.) So:

$$\frac{\textit{shorter leg of small } \triangle}{\textit{shorter leg of big } \triangle} = \frac{\textit{longer leg of small } \triangle}{\textit{longer leg of big } \triangle}$$

Filling in what we know, and calling "shorter leg of small $\triangle$ " simply "s", this becomes:

$$\frac{s}{4} = \frac{3}{6}$$

A little cross-multiplication and we can solve this, no problem:

$$\rightarrow 6 \times s = 4 \times 3$$

$$\rightarrow 6s = 12$$

$$\rightarrow \boldsymbol{s = 2}$$

Great! So the radius that the water makes inside the cup is 2 inches. Now we can solve the rest of this – it wants the volume of water in a cone shape that is 3 inches tall with a radius of 2, right? So the Base area is just: $\pi r^2 = \pi(2)^2 = \boldsymbol{4\pi}$. We know $h = 3$, and that means the volume is given by:

$$\text{Volume of Cone-Shaped Water} = \frac{1}{3}Bh = \frac{1}{3}(4\pi)(3) = \boldsymbol{4\pi}$$

Wait, how can the area of the Base and the volume of the water be the same? Because they are NOT same! They have TOTALLY different units (square inches and cubic inches) – it's like comparing apples to oranges! But even more strange, trust me. ☺

Answer:

Part a: Volume of cup = 32π in.^{3}

Part b & c: radius = 2 in; Volume of water = 4π in.^{3}

Pretty crazy that when the height is *half* as much as the total cup, there's only *one eighth* as much water! (32π in.^{3} vs. 4π in.^{3}!)

And YOU, my dear, have just finished *Girls Get Curves*. CONGRATULATIONS!! You deserve a little splurge, oh yes you do. ☺