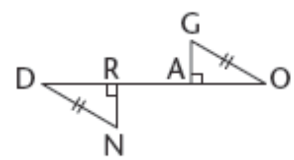
Girls Get Curves

Solution Guide for Chapter 9

Here are the solutions for the "Doing the Math" exercises in *Girls Get Curves*!

DTM from p. 151-153

2. Write out the DRAGON proof as a two-column proof, using the Givens and diagram from p. 146. *(First, try it without peeking at our strategy!)*



So, the problem from p. 146 was:

Given: In the DRAGON diagram, R & A trisect $\overline{DO}$, $\overline{DN} \cong \overline{OG}$, and $\angle DRN$ & $\angle OAG$

are right angles. Prove: $\angle D \cong \angle O$.

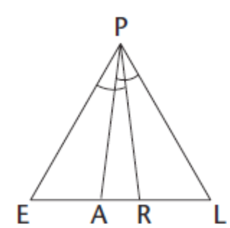
Read p. 146 to review the strategy – it's exactly the same!
Here's how it looks as a two-column proof…

♥ Proof ♥

Statements	**Reasons**
1. $\angle DRN$ & $\angle OAG$ are right angles	1. Given (This will allow us to use HL)
2. $\overline{DN} \cong \overline{OG}$	2. Given (Congruent hypotenuses – gimmie an "H"!)
3. R & A trisect $\overline{DO}$	3. Given
4. $\overline{DR} \cong \overline{AO}$	4. Definition of trisect: The two points divide a segment into three $\cong$ segments. (Congruent corresponding legs: Gimmie an "L"!)
5. $\triangle DRN \cong \triangle OAG$	5. HL (2, 4)
6. $\therefore \angle D \cong \angle O$	6. CPCTC

Ta-da! If any of that was confusing, just read the explanation on p.146 of *Girls Get Curves*. ☺

3. For PEARL, Given: $\angle EPR \cong \angle LPA$, and $\triangle EPL$ is an isosceles triangle with base $\overline{EL}$. Prove: $\angle EAP \cong \angle LRP$



So first, find the two angles we're trying to prove are congruent to each other. Got em? Good! Notice that if we could first prove that the two "outer" triangles are congruent to each other ($\triangle EAP$ & $\triangle LRP$), then we could prove that $\angle EAP$ & $\angle LRP$ with CPCTP. Let's do it!

Hm. If $\triangle EPL$ is an isosceles triangle with base $\overline{EL}$, that automatically tells us that its legs are congruent: $\overline{PE} \cong \overline{PL}$, and that is base angles are congruent: $\angle E \cong \angle L$. And those happen to be corresponding sides *and* angles in the two outer triangles. Niiiice. (Gimmie an "S"! Gimmie an "A"!)

We still need another "S" or "A." But we haven't used our first Given yet: $\angle EPR \cong \angle LPA$, which are those marked, overlapping angles up top. Those angles aren't in our desired triangles, but we can use the Subtraction property to subtract off the center angle from both, which will result in two smaller congruent angles: $\angle EPA \cong \angle LPR$. And those ARE inside the triangles we're trying to prove are congruent! (Gimmie an "A"!) Since the S is between the two A's, we'll use ASA.

Pant, pant!

Now that we have two congruent triangles, $\triangle EAP \cong \triangle LRP$, we can see that CPCTC guarantees $\angle EAP \cong \angle LRP$.

Answer:

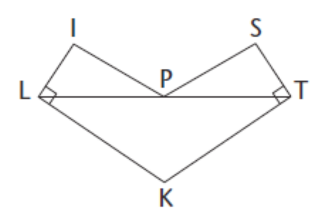
♥ Proof ♥

Statements	**Reasons**
1. $\triangle EPL$ is an isosceles triangle with base $\overline{EL}$.	1. Given
2. $\overline{PE} \cong \overline{PL}$	2. Definition of isosceles triangle: If a triangle is isosceles, then its legs are $\cong$. (Gimmie an "S"!)
3. $\angle E \cong \angle L$	3. If sides, then angles. (Gimmie an "A"!)
4. $\angle EPR \cong \angle LPA$	4. Given

5. $\angle EPA \cong \angle LPR$	5. Subtraction Property: When an angle ($\angle APR$) is subtracted from two $\cong$ angles, the differences are $\cong$. (Gimmie an "A"!)
6. $\triangle EAP \cong \triangle LRP$	6. ASA (3, 2, 5)
7. $\therefore \angle EAP \cong \angle LRP$	7. CPCTC

Note: In step 3 above, we also could have used this Reason: **Definition of isosceles triangle: If a triangle is isosceles, then its base angles are $\cong$. (Gimmie an "A"!)** But I like how nice & short the "If sides, then angles" looks. ☺

4. For LIPSTK, Given: $\overline{IP} \cong \overline{SP}$; $\angle I \cong \angle S$; $\angle ILK$ & $\angle STK$ are right angles; $\overline{LK} \cong \overline{TK}$. Prove: P is the midpoint of $\overline{LT}$. *(Hint: What does $\overline{LK} \cong \overline{TK}$ tell us about the angles in the diagram?)*



Let's not panic! We'll work backwards a bit: In order to prove that P is the midpoint of $\overline{LT}$, let's think about the definition of *midpoint*, and we realize that if we first prove that $\overline{LP} \cong \overline{TP}$, then we'd just be one step away from proving P is a midpoint, right? So, how can we prove that $\overline{LP} \cong \overline{TP}$? Well if we could first prove $\triangle LIP \cong \triangle TSP$, then CPCTC would tell us that $\overline{LP} \cong \overline{TP}$. Great! Our new goal is to prove $\triangle LIP \cong \triangle TSP$.

Hm, we're given that $\overline{IP} \cong \overline{SP}$, so that's an "S"! And we're also given that $\angle I \cong \angle S$, so that's an "A"! Hm… how we get a last "S" or "A"?

Well, we're given those two right angles, so maybe they'll help us get $\angle ILP \cong \angle STP$ somehow – which would then give us SAA. But how do we get $\angle ILP \cong \angle STP$? Well, we could use the subtraction property to get $\angle ILP \cong \angle STP$ from those right angles, but we'd need to first prove that the lower angles making up those right angles were congruent: $\angle TLK \cong \angle LTK$. Can we get those? Sure! After all, they are the base angles of that big upside-down triangle, and we know that big upside-down triangle is isosceles because we're told that its legs are congruent: $\overline{LK} \cong \overline{TK}$. (That's what the hint was getting at: those congruent segments tell us the big upside-down triangle is isosceles, which then must have congruent base angles: $\angle TLK \cong \angle LTK$.)

Ok – to recap – here's our strategy: We will prove the two upper triangles are congruent with AAS: we are given an "S" and an "A", and the way we'll get our final "A" will be to first prove the big upside-down triangle is isosceles, get the base angels congruent, and then use the Subtraction property to get $\angle ILP \cong \angle STP$. And that's our final "A"! Now SAA tells us the two upper triangles are congruent. And CPCTC tells us that their corresponding sides must be congruent, for example, $\overline{LP} \cong \overline{TP}$, and the definition of "midpoint" then tells us that *P* must be the midpoint of $\overline{LT}$. Phew! Let's write it out:

Answer:

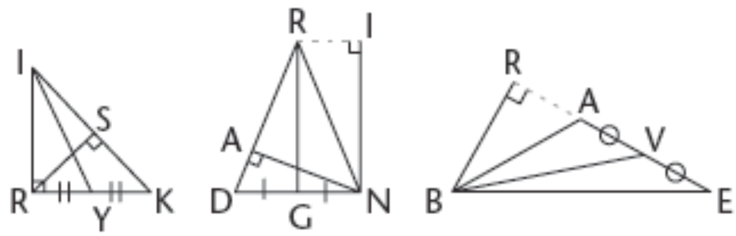
♥ Proof ♥

Statements	**Reasons**
1. $\overline{IP} \cong \overline{SP}$	1. Given (Gimmie an "S"!)
2. $\angle I \cong \angle S$	2. Given (Gimmie an "A"!)
3. $\overline{LK} \cong \overline{TK}$	3. Given (and this tells us the big upside-down triangle is isosceles!)
4. $\angle TLK \cong \angle LTK$	4. If sides, then angles
5. $\angle ILK$ & $\angle STK$ are right angles	5. Given
6. $\angle ILK \cong \angle STK$	6. If two angles are both right angles, then they are congruent to each other (duh).
7. $\angle ILP \cong \angle STP$	7. Subtraction Property: When two $\cong$ angles are subtracted from two $\cong$ angles, the differences are $\cong$. (Gimmie an "A"!)
8. $\triangle LIP \cong \triangle TSP$	8. SAA (1, 2, 7)
9. $\overline{LP} \cong \overline{TP}$	9. CPCTC
10. $\therefore$ P is the midpoint of $\overline{LT}$.	10. Definition of midpoint: If a point divides a segment into two $\cong$ segments, then it is the midpoint.

Pant, pant! Great job following that!

DTM from p. 160-161

2. Name the three medians appearing in $\triangle RIK$, $\triangle DRN$, and $\triangle BAE$.



A median is a segment going from a vertex to the midpoint of the opposite side, so we'll look for *congruent segments* to help us find the midpoints, and then the medians. Make sense? Let's do it! In RISKY, Y is a midpoint, so $\overline{IY}$ is a median. In DARING, G is a midpoint, so $\overline{RG}$ is a median, and in BRAVE, V is a midpoint, so $\overline{BV}$ is a median.

Answer: $\boldsymbol{\overline{IY}}$, $\boldsymbol{\overline{RG}}$, $\boldsymbol{\overline{BV}}$

3. Name the five altitudes appearing in $\triangle RIK$, $\triangle DRN$, and $\triangle BAE$.

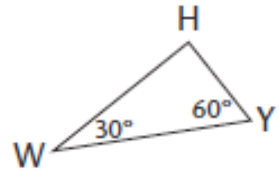
(see above for diagram)

An altitude is a segment going from a vertex of a triangle, making a right angle with the opposite (sometimes extended) side.

In RISKY, $\overline{RS}$ does that, and so do $\overline{IR}$ and $\overline{KR}$! In DARING, $\overline{NA}$ definitely does that. Does $\overline{IN}$? Nope! It starts at N, and it makes a right angle with *something*… but that dotted line isn't extending a side of this triangle at all, is it? (Very sneaky, I know.) In BRAVE, $\overline{BR}$ seems like an altitude, and indeed, that dotted line does extend the a side of the triangle, so $\overline{BR}$ <u>is</u> an altitude!

Answer: $\boldsymbol{\overline{RS}}$, $\boldsymbol{\overline{IR}}$, $\boldsymbol{\overline{KR}}$, $\boldsymbol{\overline{NA}}$, $\boldsymbol{\overline{BR}}$

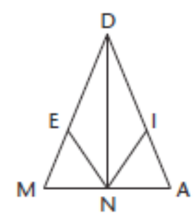
4. In $\triangle WHY$, is $\overline{HY}$ an altitude? If so, why?



Since the three angles of a triangle always add up to 180°, we know that $\angle H = 90°$, because after all: 30° + 60° + 90° = 180°, right? Well, if $\angle H = 90°$, that means $\angle H$ is a right angle, and since $\overline{HY}$ is a segment going from Y to make a right angle with the opposite side of the triangle ($\overline{WH}$), it is indeed an altitude!

Answer: Yes, $\overline{HY}$ is an altitude, because $\angle H = 90°$, so that means $\overline{HY} \perp \overline{WH}$.

5. In MEDIAN, Given: $\overline{DN}$ is a median in the isosceles triangle $\triangle MDA$ with base $\overline{MA}$, and $\angle ENM \cong \angle INA$. Prove: $\overline{EN} \cong \overline{IN}$.



This sure feels like a CPCTC proof – we need to prove that two segments are congruent, in a diagram that has all sort of triangles in it! So if we can find two triangles that have $\overline{EN}$ & $\overline{IN}$ as corresponding sides, and if we can prove those triangles are congruent, then we'll be golden!

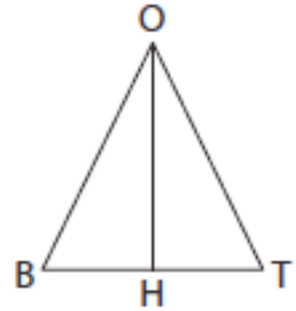
Hm… which two triangles could these be? Well, it could be those top, long triangles ($\triangle DEN$ & $\triangle DIN$) or the two smaller, lower ones ($\triangle MEN$ & $\triangle AIN$) – they both have $\overline{EN}$ & $\overline{IN}$ as corresponding sides. Well, which ones are easier to prove congruent? We're told that $\overline{DN}$ is a median in the isosceles triangle $\triangle MDA$ - that's a lot of information right there! You see, since $\overline{DN}$ is a median, that means $\overline{MN} \cong \overline{NA}$, which are corresponding sides for $\triangle MEN$ & $\triangle AIN$, right? Let's focus on that smaller pair of triangles.

$\overline{MN} \cong \overline{NA}$ means we just got an "S"! And since $\triangle MDA$ is isosceles, that means its base angles are congruent: $\angle M \cong \angle A$. (Gimmie an "A"!) And then we're told flat out that $\angle ENM \cong \angle INA$, which gives us our last "A"! We have ASA for $\triangle MEN$ & $\triangle AIN$, which allows us to use CPCTC to prove that a set of corresponding sides must be congruent: $\overline{EN} \cong \overline{IN}$. Let's write it out:

♥Proof♥

Statements	**Reasons**
1. $\triangle MDA$ is an isosceles triangle	1. Given
2. $\angle M \cong \angle A$	2. If a $\triangle$ is isosceles, then its base angles are $\cong$. (Gimmie an "A"!)
3. $\overline{DN}$ is a median	3. Given
4. $\overline{MN} \cong \overline{NA}$	4. Definition of median (converse) (Gimmie an "S"!)
5. $\angle ENM \cong \angle INA$	5. Given (Gimmie an "A"!)
6. $\triangle MEN \cong \triangle AIN$	6. ASA (2, 4, 5)
7. $\therefore \overline{EN} \cong \overline{IN}$	7. CPCTC

Great job!



6. In BOTH (see above), Given: $\triangle BOT$ is an isosceles triangle with base $\overline{BT}$, and $\overline{OH}$ is an altitude. Prove: $\overline{OH}$ is a median. *(Hint: Try doing it without peeking, but see p. 157 for the logic if you need it! And if you want your answer to match my online solution, use HL instead of SAA.)*

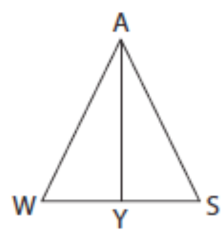
The logic is totally spelled out on p. 157 – and here's the actual proof, in two-column form!

♥Proof♥

Statements	**Reasons**
1. $\triangle BOT$ is an isosceles triangle with base $\overline{BT}$	1. Given
2. $\overline{BO} \cong \overline{TO}$	2. If a $\triangle$ is isosceles, then its legs are $\cong$. (Gimmie an "H"!)
3. $\overline{OH}$ is an altitude	3. Given
4. $\angle OHB$ & $\angle OHT$ are right angles.	4. An altitude creates right angles with its opposite side ($\overline{BT}$). (Now we'll be allowed to use HL, since we

	have right angles.)
5. $\overline{OH} \cong \overline{OH}$	5. Reflexive Property (Gimmie an "L"!)
6. $\triangle BOH \cong \triangle TOH$	6. HL (2, 5) Yay, we have $\cong$ triangles!
7. $\overline{BH} \cong \overline{TH}$	7. CPCTC
8. H is the midpoint of $\overline{BT}$	8. Definition of *midpoint*: If a point divides a segment into two congruent segments, then it's the midpoint.
9. $\therefore \overline{OH}$ is a median.	9. Definition of *median*: If a seg connects a vertex (O) to the midpoint of the opposite side (H), then it's the median.

7. In WAYS, Given: $\triangle WAS$ is an isosceles triangle with base $\overline{WS}$, and $\overline{AY}$ is a median. Prove: $\overline{AY}$ is an altitude. *(Hint: Use the equidistant stuff from p. 158 and the definition of* altitude.*)*



How do we prove something is an altitude? Well, we have to prove that it goes from one vertex (which we can indeed assume from the diagram and doesn't even have to be mentioned in the proof!), and that it's perpendicular to the (sometimes extended) opposite side. In this diagram, $\overline{AY}$ touches the opposite side ($\overline{WS}$), so our goal will be to prove that it creates a right angle with that side!

Hm, how do we do that? Well, on p. 158, the second equidistant theorem that can give us perpendicular lines – we just have to prove that there are two points that are each equidistant to the segment's endpoints. Here, in order to prove that $\overline{AY} \perp \overline{WS}$ (in fact the theorem does more – it proves that $\overline{AY}$ is the perpendicular bisector of $\overline{WS}$), we'd need to show that *A* is equidistant to *W* & *S*, and that *Y* is also equidistant to *W* & *S*. In other words, we'd need to show that $\overline{AW} \cong \overline{AS}$ and $\overline{WY} \cong \overline{SY}$.

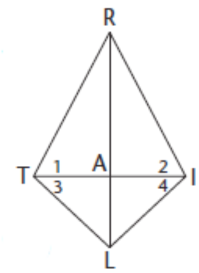
No problem! We've been told that $\triangle WAS$ is isosceles, so its legs must be congruent: $\overline{AW} \cong \overline{AS}$. And we know $\overline{WY} \cong \overline{SY}$ because we've been told that $\overline{AY}$ is a median, so that means *Y* is the midpoint and so it follows that $\overline{WY} \cong \overline{SY}$. And these two sets of congruent segments tell us we have two equidistant points we were looking for, in order to use the second equidistant theorem from p. 158, and get perpendicularity.

Let's write it out!

♥Proof♥

Statements	**Reasons**
1. $\triangle WAS$ is an isosceles triangle with base $\overline{WS}$	1. Given
2. $\overline{AW} \cong \overline{AS}$	2. If a triangle is isosceles, then its legs are congruent.
3. $\overline{AY}$ is a median	3. Given
4. $\overline{WY} \cong \overline{SY}$	4. Definition of median
5. $\overleftrightarrow{AY} \perp$ bis. $\overline{WS}$	5. If two points (*A* & *Y*, from steps 2 & 4) are each equidistant to the endpoints of a seg ($\overline{WS}$), then those two points determine the seg's $\perp$ bisector.
6. $\overleftrightarrow{AY} \perp \overline{WS}$	6. Definition of $\perp$ *bisector* (A $\perp$ bisector is $\perp$ to its segment.)
7. $\therefore \overline{AY}$ is an altitude	7. Definition of *altitude* (If a seg comes from a vertex – assumed from diagram – and is $\perp$ to the opposite side, $\overline{WS}$, then it's the altitude).

8. In TRAIL, Given: $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. Prove: $\overline{TA} \cong \overline{IA}$. *(Hint: First prove $\overleftrightarrow{RL}$ is a $\perp$ bis; see pp. 158–9.)*



Let's take the hint and first prove that $\overleftrightarrow{RL}$ is the perpendicular bisector (of $\overline{TI}$). How do we do that? We need to find two points that are each equidistant to the endpoints of $\overline{TI}$ (*T* & *I*). If we can find the correct congruent segments, we'll have done the trick. For example, if we prove that $\overline{TR} \cong \overline{IR}$, then we know *R* is equidistant to *T* & *I*.

And we can! Since we've been told that $\angle 1 \cong \angle 2$, we know that $\triangle TRI$ is an isosceles triangle, and "if angles, then sides" gives us $\overline{TR} \cong \overline{IR}$. Great! How do we get another point that's equidistant to *T* & *I*? Well, let's do the same exact thing with the "upside-down" triangle, $\triangle TLI$. Since we're told that $\angle 3 \cong \angle 4$, we can conclude that $\overline{TL} \cong \overline{IL}$. Then we can use the second theorem on p. 158 to prove that $\overleftrightarrow{RL} \perp$ bis. $\overline{TI}$, and then because of the "bisector" part of "perpendicular bisector," we know that if $\overline{TI}$ is bisected at the point *A*, it must be true that $\overline{TA} \cong \overline{IA}$. Nice.

♥Proof♥

Statements	**Reasons**
1. $\angle 1 \cong \angle 2$	1. Given
2. $\overline{TR} \cong \overline{IR}$	2. If angles, then sides
3. $\angle 3 \cong \angle 4$	3. Given
4. $\overline{TL} \cong \overline{IL}$	4. If angles, then sides
5. $\overleftrightarrow{RL} \perp$ bis. $\overline{TI}$	5. If two points (*R* & *L*, from steps 2 & 4) are each equidistant to the endpoints of a seg ($\overline{TI}$),

	then those two points determine the seg's $\perp$ bisector.
5. $\therefore \overline{TA} \cong \overline{IA}$	5. If a point (A) is on the $\perp$ bisector of a segment ($\overline{TI}$), then that point bisects the segment into two congruent halves.

Great job with that chapter!