Girls Get Curves

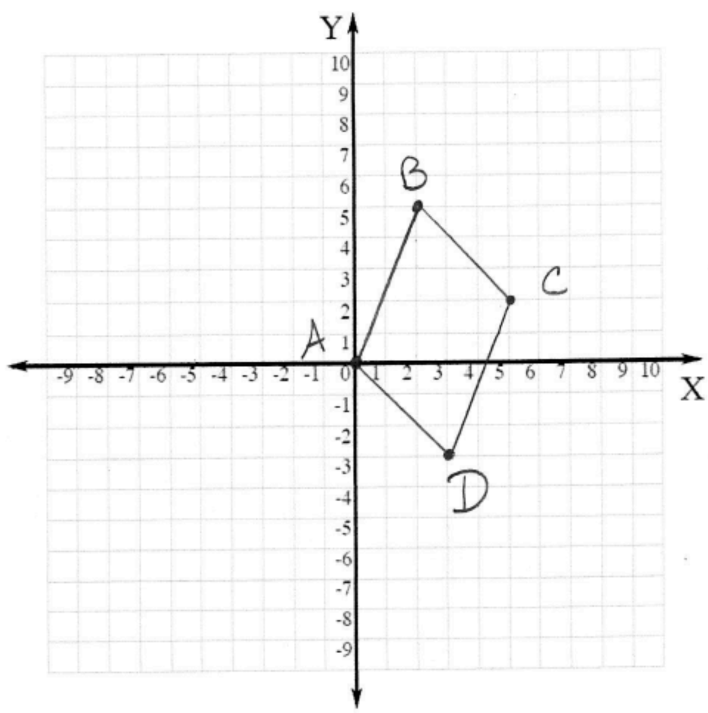
Proving Two Sides of a Quadrilateral are Parallel – On the Coordinate Plane

(As promised on p. 267 in *Girls Get Curves*)

In chapter 16, we saw that one way to prove that a quadrilateral is a parallelogram is to prove that both sets of opposite sides are parallel.

Sometimes we're given a quadrilateral's coordinate points, and here's how to prove it's a parallelogram in that case: In chapter 11 in *Hot X: Algebra Exposed*, we learned how to determine if two lines are parallel – by seeing if their slopes are equal. And that's what we can do here, too…

Here's how it works: Let's say we have a quadrilateral defined by the four points $A = (0, 0)$, $B = (2, 5)$, $C = (5, 2)$, and $D = (3, -3)$.



If we can show that $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$, then we'll have proven that *ABCD* is a parallelogram. Let's do it!

First we need to find the slope of all four sides. For each side, we just pick one point to be "point #1" and the other point to be "point #2," and then the slope between (x_1, y_1) and (x_2, y_2) can be expressed as:

$$\text{Slope} = \frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x} = \frac{y_1 - y_2}{x_1 - x_2}$$

See p. 157 in Hot X: Algebra Exposed to review that. (And to see how lounge chairs and playing favorites can make this easier to remember…)

Ready? Let's do it!

So… what's the slope of $\overline{AB}$? We'll use (2, 5) as point #1 and (0, 0) as point #2:

$$\text{Slope of } \overline{AB} = \frac{rise}{run} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{5-0}{2-0} = \mathbf{\frac{5}{2}}$$

Now we know the slope of one side of the quadrilateral. Time for the other sides!

To find the slope of $\overline{BC}$, let's use (5, 2) as point #1 and (2, 5) as point #2:

$$\text{Slope of } \overline{BC} = \frac{rise}{run} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2-5}{5-2} = \frac{-3}{3} = \mathbf{-1}$$

To find the slope of $\overline{CD}$, let's use (5, 2) as point #1 and (3, –3) as point #2:

$$\text{Slope of } \overline{CD} = \frac{rise}{run} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{2-(-3)}{5-3} = \mathbf{\frac{5}{2}}$$

To find the slope of $\overline{AD}$, let's use (3, –3) as point #1 and (0, 0) as point #2:

$$\text{Slope of } \overline{AD} = \frac{rise}{run} = \frac{y_1 - y_2}{x_1 - x_2} = \frac{-3-0}{3-0} = \frac{-3}{3} = \mathbf{-1}$$

How about that! The slope of $\overline{AB}$ is the same as the slope of $\overline{CD}$ (they both equal $\mathbf{\frac{5}{2}}$), which by definition means that $\overline{AB} \parallel \overline{CD}$. And since the slope of $\overline{BC}$ and the slope of $\overline{AD}$ are equal (they both equal **–1**), that means $\overline{AD} \parallel \overline{BC}$.

Since we've shown that both sets of opposite sides of the quadrilateral are parallel, we've now proven that ***ABCD* is a parallelogram**!

∴ ***ABCD* is a parallelogram**

And that's how it works. Not so bad, right?