

Two More Methods for Proving That Triangles Are Similar

(As promised in the footnote of p. 293 in Girls Get Curves)

In chapter 17 of *Girls Get Curves*, we saw how to prove that two triangles are similar with the AA shortcut – see p. 289 to review. Well, there are actually two other ways to prove that triangles are similar. They're called **SSS~** and **SAS~**, and here they are!

SSS~

If the ratios of all three corresponding sides of two triangles are equal, then the two triangles are similar.

So like on p.286 of *Girls Get Curves*, if we were just told that $\frac{OA}{RY} = \frac{AT}{YE} = \frac{OT}{RE}$, we wouldn't have to be told anything about the angles; we'd automatically know that

 $\triangle OAT \sim \triangle RYE$.

SAS~

If the ratios of *two* corresponding sides are equal, and if the included angles are congruent, then the two triangles are similar.

So in the below triangles, since $\angle E \cong \angle Y$, and it's totally true that $\frac{2}{3} = \frac{10}{15}$, we know that

 $\triangle PIE \sim \triangle SKY$.



Those look a little like pieces of pie floating around in the sky, don't you think?

Notice the little wavy "similar" symbol on SSS~ and SAS~. We need it there to keep from confusing them with the shortcuts for proving triangles congruent, SSS and SAS.

CASTC and CSSTP

And just when we thought there were no more letter combinations to learn... there are!

These are related to the good 'ol CPCTC (Corresponding Parts of Congruent Triangles are Congruent), which is that thing at the bottom of many geometry proofs.

Unless your teacher insists on it, I wouldn't bother memorizing the actual letter combinations "CASTC" and "CSSTP," because it's really annoying. Instead, think about what they mean; they're *just* like CPCTC, except for similar triangles, and also, one is for angles and one is for sides. (In fact, they are the converses of the two properties of similar triangles listed on p. 286.)

Check 'em out:

CASTC: Corresponding Angles of Similar Triangles Are Congruent. So if we found out that two triangles are similar (like with SSS ~, maybe?) then we'd

automatically know that all the corresponding angles are congruent.

CSSTP: Corresponding Sides of Similar Triangles are Proportional.

So let's say we used AA to discover that two triangles are similar, then we know that all sets of corresponding sides are proportional to each other. This Rule officially allows us to, for example, set up the proportions between the sides' ratios like we did on p. 290 to find missing sides. No biggie.

Step By Step

Proofs involving Similar Triangles

Step 1. Usually our job will be to prove that two triangles are similar, and then go from there. That means our first goal is to get some congruent angles and/or proportional sides, right? And keep this in mind: parallel lines might let us use the transversal (escalator!) rules from p. 218 which can give us congruent angles. Trisected or bisected segments or midpoints could give us good info on lengths, etc. Once you've figured out which two triangles are probably similar, if the orientations aren't the same, <u>draw the two triangles so they are in the same position</u> (which might mean you have to rotate or flip one!). **Step 2.** Once we have two congruent angles (AA), three proportional sets of sides (SSS~), or two proportional sets of sides and an included angle (SAS~), we've proven the two triangles similar!

Step 3. If that's what we needed to prove, we're done! Otherwise, now we'll probably use either CSSTP or CASTP to prove angles congruent or sides proportional, and go from there.

Step 4. Make sure you've proven what the problem was asking for. Done!



As I mentioned in *Girls Get Curves*, most of the time, we'll only need AA and CSSTP or CASTC. The SSS ~ and SAS ~ shortcuts just don't come up that often. But we'll do a couple of examples anyway, because we're just special like that.

Hey, have you ever set up a camping tent? It's not my, um, strong suit. My tents usually end up looking a little like the below diagram.¹

See the CAMPING diagram to the right.

Given: A & M trisect \overline{CP} , N & I trisect \overline{GP} , AN = 4 and CG = 6. Do a paragraph proof to explain why $\triangle CPG \sim \triangle APN$.



Step 1. Ok, so $\triangle CPG$ is the great big triangle, and $\triangle APN$ is the

smaller one, nestled inside it. Their orientations match up, which is

nice – no need to rotate and redraw one of them just to see what's going on. Since AN = 4and CG = 6, we can figure out the ratio of those corresponding sides, right? It's just:

 $\frac{AN}{CG} = \frac{4}{6} = \frac{2}{3}$. And if we can prove that the other corresponding sides' ratios are also $\frac{2}{3}$, we can use SSS ~ to prove that the triangles are similar! Sounds like a plan.

We've been given that A & M trisect \overline{CP} , which means that $\underline{CA} = \underline{AM} = \underline{MP}$, right? Well gosh, if those little segments are all equal to each other, and *two* of them are used for that side of our small triangle, and *three* are used for the big triangle, that sure seems like a ratio of 2:3! Here's how we can be more math-y about it (which teachers really love). We'd love to show that $\frac{AP}{CP} = \frac{2}{3}$, right? Let's write *AP* and *CP in terms of* one of the little segments, MP.²

Notice on the diagram that AM + MP = AP. Since trisection gave us <u>AM = MP</u>, we can use substitution to say MP + MP = AP; in other

¹ The diagram doesn't look anything like a tent, does it? Now you're getting my point. ² To brush up on writing expressions *in terms of* a variable, check out chapter 6 in *Hot X: Algebra Exposed!*

words: 2MP = AP. (Read that till it makes sense.) Also, notice on the diagram that CA + AM + MP = CP, and since $\underline{CA} = AM = MP$, we can say 3MP = CP. And now, the ratio of those bottom corresponding sides can be written as: $\frac{AP}{CP} = \frac{2MP}{3MP} = \frac{2}{3}$. Remember, MP is just some length – it's a positive number whose value we don't happen to know, but we can certainly cancel it from the top and bottom of that fraction. Cool trick, huh? The same *exact* strategy works for the leftmost corresponding sides, and we get that $\frac{PN}{PG} = \frac{2}{3}$. **Steps 2-4.** And how about that? We've now shown that all three sets of corresponding sides' ratios are equal: $\frac{AN}{CG} = \frac{AP}{CP} = \frac{PN}{PG}$, because they all equal $\frac{2}{3}$, after all! And by $SSS \sim$, we now have similar triangles: $\triangle CPG \sim \triangle APN$. That's what we were supposed to

prove, and we're done!





The Reflexive property works for angles, too. In fact, since the two triangles above in CAMPING share $\angle P$, we could have used SAS ~ instead of SSS ~ to get similarity.

Let's see a couple more examples, this time keeping our new friends in mind, CASTC and CSSTP... In the STARZ diagram with the lengths given, do a paragraph

proof to explain why $\overline{ST} \parallel \overline{RZ}$.



Remember the VIOLET problem from p. 295? We'll use a similar³ logic, but in reverse order: We'll use the given lengths

to prove similar triangles, which will prove congruent angles, which will then give us parallel lines!

Notice that since $\frac{1}{3} = \frac{2}{6}$, we can say: $\frac{SA}{AZ} = \frac{TA}{AR}$. Nice.

Also, the vertical angles on the diagram tell us that $\angle SAT \cong \angle ZAR$, so by SAS ~, we've shown that $\triangle SAT \sim \triangle ZAR$, and let's double check the correspondence by drawing them separately, to make sure SA corresponds with AZ, etc. Yep!



Great! Now, CASTC tells us that corresponding angles are congruent, so we know that for example, $\angle STA \cong \angle ZRA$, right? And extending the lines $\overline{ST} \& \overline{RZ}$, it looks like the mall, and focusing on \overline{RT} as the only escalator (transversal),



³ No pun intended.

 $\angle STA \& \angle ZRA$ end up being congruent alternate interior angles. See the big sideways Z? Finally, the rule "If alt int \angle 's are \cong , then lines are ||" from p. 230 tells us that $\overline{ST} || \overline{RZ}$, and finishes off our paragraph proof. Nice!



One more example – this time a full-blown two-column proof...

In the CASTLE diagram, Given: $\triangle CSL$ is an isosceles triangle, And $AC \cdot TE = AE \cdot TL$. Prove: $\angle CAE \cong \angle LTE$.



Okay, what's our strategy? In order to prove that $\angle CAE \cong \angle LTE$, it would sure be great if we could first prove that $\triangle CAE \sim \triangle LTE$, right? Then those two angles would be corresponding angles on two similar triangles, and we'd just use CASTC to prove they are congruent. How can we prove that $\triangle CAE \sim \triangle LTE$? Let's see what we're given. Hm - $AC \cdot TE = AE \cdot TL$ is one of those tricky ways of writing a ratio; if we're clever, we can see that this being true is the equivalent statement as $\frac{AC}{TL} = \frac{AE}{TE}$. In fact, if we take the cross-product of this fraction equation, we actually get $AC \cdot TE = AE \cdot TL$. But how do we get from this statement to the fraction equation? <u>By dividing both sides by *TL* and then</u> <u>both sides by *TE*</u>. Don't worry if you wouldn't have thought of it on your own! But now you have this trick under your belt: If you see two equal products in a triangle proof, there's a good chance it's the equivalent statement to a ratio of sides, so it's worth investigating.

Okay, so now we know that
$$\frac{AC}{TL} = \frac{AE}{TE}$$
, which means that we have two sets of

corresponding sides with equal ratios. If we had a third such side, then we'd have SSS~, right? But we don't... hm. Can we use SAS~ somehow? Yep! That other given – the fact that $\triangle CSL$ is an isosceles triangle, means that we know $\angle C \cong \angle L$, and that gives us SAS~. (Notice that our similar triangles don't have the same orientation, but we could draw them that way if we really wanted, just by flipping one of them.)

Cool – now we know that $\triangle CAE \sim \triangle LTE$, and that means all their corresponding angle pairs must be congruent – including $\angle CAE \cong \angle LTE$. Let's do this!



Statements	Reasons
1. $\triangle CSL$ is an isosceles	1. Given
triangle	
2. $\angle C \cong \angle L$	2. Alternate definition of isosceles triangle. (Gimmie
	an "A"!)
3. $AC \cdot TE = AE \cdot TL$	3. Given

$4. \ \frac{AC}{TL} = \frac{AE}{TE}$	4. Algebra: Divide both sides of $AC \cdot TE = AE \cdot TL$ by <i>TL</i> and then by <i>TE</i> . (Great! Now 2 sets of corresponding ratios are equal!)
5. $\triangle CAE \sim \triangle LTE$	5. SAS~ (Similar triangles!)
6. $\angle CAE \cong \angle LTE$	6. CASTC: Corr. \angle 's \triangle of sim. are \cong .

This stuff ain't easy! I'm so proud of you for reading through that. If it's not quite sinking in, try reading chapter 17 again and then reading this again. You can do it!